Loving or Fighting Inflation: Managing debt dynamics

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Abstract

The economic landscape post-COVID-19 and the energy crisis has underscored the critical role of government intervention in cushioning against shocks and aiding economic recovery. Focusing on the United Kingdom, we analyze optimal fiscal and monetary policies in scenarios marked by differing debt-to-GDP ratios, notably comparing the low ratio of 2007 to the high ratio of 2021. By exploring dynamic responses to shocks and the utilization of policy tools like taxation, public debt, and inflation, we offer insights into effective strategies for navigating economic uncertainties. Specifically, we highlight how higher debt-to-GDP ratios necessitate nuanced approaches to managing public debt in response to shocks. Additionally, we find that in the absence of fiscal tools, inflation can serve as an effective adjustment mechanism, suggesting that accepting moderate inflation may be optimal in certain scenarios. We use an extended Lagrangian approach for analytical results and a truncated representation of incomplete markets model for quantitative findings, offering a manageable framework for studying policy dynamics in response to economic shocks.

Keywords: Heterogeneous agents, Ramsey policies, Monetary policy, Fiscal Policy, Inflation, Public Debt.
JEL Codes: E21, E44, E61, E62, E32, D31, D52, H21.

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1 Introduction

The evolving economic landscape, exacerbated by events like COVID-19 and energy crises, underscores the critical role of government intervention in cushioning against shocks and fostering economic recovery. As countries contemplate additional public spending and debt issuance to address these challenges, significant shifts in fiscal structures are evident, as exemplified by the case of the United Kingdom.

Over the period between 2007 and 2021, the United Kingdom has experienced significant transformations in both the size of the state and income inequalities. In 2007, the UK exhibited a lower public debt-to-GDP ratio at 43.1%, alongside moderate income inequality reflected by a Gini coefficient of 38.6 post-taxation. By 2021, however, the landscape has dramatically shifted, with the UK's public debt-to-GDP ratio more than doubling to 101%, indicating a larger state role in the economy. Interestingly, this period also witnessed a substantial reduction in income inequality, with the Gini coefficient dropping from 38.6 to 29.9, as shown in Table 1. Additionally, recent years have witnessed an increase in inflation, posing challenges for central banks striving to maintain control, as illustrated in Figure 1.

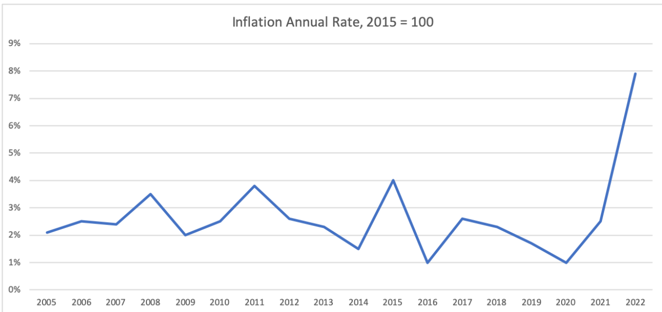


Figure 1: Annual Consumer Price Index (CPI) inflation rate, encompassing all items and indexed with a base year of 2015 set at 100. Source: Office for National Statistics.

To address these shifts and their implications, this paper delves into the analysis of optimal monetary and fiscal policy in heterogeneous-agent models, considering two distinct scenarios. One scenario represents a low debt-to-GDP ratio akin to the UK in 2007, while the other mirrors the high debt-to-GDP ratio observed in 2021. By scrutinizing these scenarios, we aim to discern whether recent changes in fiscal policy landscapes represent a shift in optimal responses to shocks and whether the current fiscal policy necessitates acceptance of higher inflation scenarios.

	B/Y	Gini bef.	Gini aft.
United Kingdom 2007	43.1	53.5	38.6
United Kingdom 2001	101	50.2	29.9

Table 1: Summary of the size of the state and income inequalities in the United Kingdom in 2007 and 2021.

To examine the optimal dynamic responses, we undertake a structured approach. Firstly, we construct calibrated models reflecting the economic conditions of the United Kingdom in 2007 and 2021. We chose 2007 as it predates the financial crisis and the COVID-19 pandemic, pivotal events that reshaped fiscal structures. Conversely, 2021 represents the post-COVID-19 and energy crisis landscape. We acknowledge that extended periods of high debt may signal a new normal, implying a potential shift to a different steady state, a critical consideration in our analysis.

Our model encompasses various elements including capital accumulation, labor tax progressivity, capital tax, public debt, and inflation, with incomplete markets for idiosyncratic risk being the only friction considered. By incorporating monetary policy, we can observe how inflation and taxes are used to manage debt dynamics. This underscores the importance of inflation as a tool, especially in scenarios where fiscal tools are unavailable.

We operate under the assumption that the observed fiscal decisions and income inequalities for each of the years considered stem from the optimal choices of a benevolent planner, whose aim is to maximize intertemporal welfare in a heterogeneous-agent model. The planner is endowed with her own Social Welfare Function and comprehends the distortions and general equilibrium effects of all fiscal and monetary instruments.

Given our interest in the dynamics of the fiscal and monetary system within the quantitative model, we initially estimate a Social Welfare Function consistent with the observed United Kingdom debt-to-GDP ratio and income inequality measures highlighted earlier. Using the literature on the inverse taxation problem, we empirically estimate the Social Welfare Function, following the methodology outlined in Heathcote & Tsujiyama 2021 and Le Grand et al. 2022. This approach ensures we estimate the social weights such that the observed fiscal policy and redistributive outcomes align with the social planner's optimality condition in the steady state.

However, analyzing a model with such a wide range of fiscal and monetary policy tools poses computational challenges due to the complexity involved. To address these complexities, we adopt a sequential representation of heterogeneous-agent models and employ a truncation procedure to obtain a finite state space, as outlined in LeGrand & Ragot 2022. Subsequently, we utilize the Lagrangian approach developed by Marcet & Marimon (2019) to derive the first-order conditions of the Ramsey problem with commitment.

Once our model approximates the steady-state system of the UK in both 2007 and 2021, we analyze the optimal responses to public spending and technology shocks with varying persistence levels. By introducing transitory demand and supply shocks, we observe how variables evolve around this well-defined steady state. Our analysis reveals different optimal strategies depending on shock persistence, particularly in terms of public debt management and inflation paths. Comparing the UK economy's performance in 2021 and 2007 following various shocks sheds light on the dynamics of optimal fiscal and monetary policy. In scenarios where only fiscal policy instruments are available, distinct strategies emerge for the social planner based on shock persistence, with fiscal policy adjustments serving as the primary tool to mitigate shocks in the absence of monetary policy interventions.

For instance, in response to a positive government spending shock, the decline in capital prompts the planner to consider increasing public debt to facilitate consumption smoothing. However, the optimal response varies with shock persistence. In scenarios of low shock persistence, the planner opts to increase public debt to counteract the fall in capital. Conversely, in high-persistence scenarios, sustained capital decline makes increasing public debt more costly, leading the planner to anticipate future tax hikes instead.

The responses in 2007 and 2021 reveal differences in required tax increases to stabilize public debt, with higher levels needed in 2021 due to elevated debt levels. This underscores the evolving nature of fiscal policy adjustments over time, necessitating more adjustment for the same shock type in the current scenario.

Introducing monetary policy instruments alongside fiscal tools alters response dynamics. In this framework, the planner may increase public debt even in the face of persistent shocks in government expenses. This strategic decision stems from the need to significantly raise interest rates to maintain stable inflation, thereby mitigating the impact on capital. Consequently, increasing public debt emerges as a proactive measure to counter the adverse effects of heightened government expenses. These findings contrast with those of Ragot & Legrand 2023, who argue that the optimal path for public debt depends on shock persistence. Under optimal fiscal and monetary policy, inflation ideally remains at zero, achieved through increasing interest rates throughout the business cycle.

We posit that if the response in terms of fiscal policy parameters is more pronounced for the UK economy in 2021 compared to 2007, then the absence or restriction in the use of fiscal instruments might necessitate resorting to monetary policy tools, potentially leading to a departure from the previously assumed optimal path of zero inflation. Analyzing government expenditure shocks for the UK in 2007 demonstrates that restricting tax adjustments leads to a wealth effect, reducing labor supply and triggering a recession. Additionally, increased government spending raises public debt to stabilize the budget, regardless of shock persistence. Similar dynamics are observed when analyzing negative productivity shocks. In both cases, the planner can utilize inflation as a tool, leading to the adoption of debt deflation measures to mitigate the increase in public debt.

The analysis extends to scenarios where monetary rules are introduced, assuming fiscal tools cannot be used. Here, inflation serves as a policy tool to stimulate aggregate demand, alleviate debt burdens, and mitigate the risks associated with economic shocks. The persistence of the shock influences the optimal path of inflation, with the optimal path for inflation in 2021 being higher than in 2007.

Overall, the analysis underscores the dynamic nature of optimal fiscal and monetary policy, with inflation emerging as a crucial adjustment mechanism in response to economic shocks when fiscal tools are unavailable or restricted.

Related literature. Our paper contributes to various streams of literature. First, we contribute to the Inverse Optimal Taxation Problem, which estimates Social Welfare Function from actual fiscal policies, as demonstrated in prior studies such as Bargain & Keane 2010, Bourguignon & Amadeo 2015, Chang et al. 2018, Heathcote & Tsujiyama 2021, Le Grand et al. 2022. Furthermore, our paper is situated within the Optimal Fiscal Policy literature, aligning with works by Werning 2007 and Bassetto 2014.

Second, our research contributes to the emerging literature on optimal policies in heterogeneous-agent models. While existing studies have explored the effects of fiscal experiments in such frameworks, including those by Heathcote 2005 and Kaplan & Violante 2014, our work extends this by considering equilibrium multiplicity in an economy with capital, as well as exploring scenarios without aggregate shocks, as evidenced by Aiyagari 1995, Aiyagari & McGrattan 1998, and

Dávila et al. 2012. Moreover, we add to the literature by solving a Ramsey problem to determine optimal tax systems, following the approach of Dyrda & Pedroni 2018 and Açikgöz et al. 2018.

Furthermore, our paper intersects with the Optimal Monetary Policy literature, with references to recent discussions by Kaplan et al. 2018, Auclert 2019, Gornemann et al. 2016, and Le Grand et al. 2021 on the redistributive consequences of monetary policy. Other references in the strand of optimal monetary policy with incomplete markets can be seen in Acharya et al. 2019, Bhandari et al. 2021, and Nuno & Thomas 2019. Notably, we innovate by introducing fiscal tools and comparing their effects in the absence of fiscal mechanisms. Our work builds upon the findings of Ragot & Legrand 2023 by incorporating monetary policy considerations and demonstrating that in certain circumstances, it may be optimal to increase debt even in scenarios of high persistent shocks.

Additionally, our work is related to studies on the distributional effects of inflation, such as Doepke & Schneider 2006. Finally, our research contributes to the literature on optimal inflation paths, aligning with recent works by Nuño & Thomas 2022, McLeay & Tenreyro 2020, Blanco 2021, McKay & Wolf 2022, and Bilbiie & Ragot 2021, among others.

The structure of our paper is as follows: Section 2 presents the Model, Section 3 covers the Ramsey Problem and the conditions for obtaining the allocation under Optimal Fiscal and Monetary Policy, and Section 4 outlines the quantitative investigation and numerical approach to solving this class of problem. Finally, Section 5 concludes the paper.

2 The model

Time is discrete, indexed by $t \geq 0$. The economy is populated by a continuum of ex-ante identical agents. The population of size 1 is distributed on a segment J following a non-atomic measure ℓ: $J(\ell) = 1$. We follow Green (1994) and assume that the law of large number holds.

2.1 Risk structure

At the beginning of each period, agents face an uninsurable idiosyncratic labor productivity shock y that can take Y distinct values in the set $\mathcal{Y} \subset \mathbb{R}_+$. The productivity shock y follows a first-order Markov process with a transition matrix $(\pi_{yy'})_{y,y'}$. In each period the fraction of agents with productivity y is constant and denoted by S_y such that $S_y = \sum_{\tilde{y}} \pi_{\tilde{y}y} S_{\tilde{y}}$ for all $\tilde{y} \in \mathcal{Y}$. By normalizing the average productivity to 1 we obtain $\sum_y S_y y = 1$. The history of idiosyncratic shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\} \in \mathcal{Y}^{t+1}$, where $y_{i,\tau}$ denotes the productivity of agent i in period τ. We also denote by θ_t the measure of date-t idiosyncratic histories, that can be deduced from transition probabilities. More precisely $\theta_t(y^t)$ corresponds to the share of agents with history y^t at date t. The economy also faces in each period an aggregate risk affecting the economic productivity. This risk is denoted $(z_t)_{t\geq 0}$ and is assumed to follow an AR(1) process. The economy-wide productivity will be denoted by Z_t and is related to z_t in the following fashion $Z_t = Z_0 e^{z_t}$. Finally the history of aggregate shocks up to period t is denoted by $z^t = \{z_0, ..., z_t\} \in \mathbb{R}^{t+1}$. In this economy an agent i is allowed to adjust her labor supply, $l_{i,t}$, and earns the before-tax wage rate $\tilde{w}_t$. Therefore, her total before-tax wage amount in period t is determined by the expression $\tilde{w}_t y_{i,t} l_{i,t}$.

2.2 Preferences

Agents value streams of consumption $(c_{i,t})_{t\geq 0}$ and of labor $(l_{i,t})_{t\geq 0}$ according to a time-separable utility functions given by $\sum_{t=0}^{\infty}\beta^t U(c_{i,t}, l_{i,t})$ with $\beta \in (0,1)$. The period utility function $U(c_{i,t}, l_{i,t})$ is assumed to be separable:

$$U(c_{i,t}, l_{i,t}) = u(c_{i,t}) - v(l_{i,t}). \tag{1}$$

The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, strictly increasing, and strictly concave, with $u'(0) = \infty$, while $v : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, strictly increasing, and strictly convex, with $v'(0) = 0$.

2.3 Production

There is a continuum of monopolistically competitive firms producing a differentiated good, $Y_{j,t}$. Those goods are subsequently aggregated into a final output using an aggregator with an elasticity of substitution between varieties denoted by ε according to:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Associated with the output aggregator, the profit maximization for the firm producing the final output implies:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \text{ where } P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}.$$

The price P_t is the aggregate price index. The intermediary firms are endowed with a Cobb-Douglas production function given by $Y_{j,t} = Z_t k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}$. In equilibrium the production of firm j will equalize the demand for the product j by the final good firm, with the intermediate good being sold at the price $\frac{P_{j,t}}{P_t}$. By letting the real wage before-tax be given by $\tilde{w}_t$ and the real capital interest rate before-tax and depreciation be given by $\tilde{r}_t^k$, where depreciation is $\delta > 0$, the cost minimization problem of the firm implies:

$$\tilde{r}_t^k + \delta = \xi_{j,t}\alpha\frac{Y_{j,t}}{k_{j,t}} \quad \text{and} \quad \tilde{w}_t = \xi_{j,t}(1-\alpha)\frac{Y_{j,t}}{l_{j,t}}, \tag{2}$$

where $\xi_{j,t}$ is the Lagrange-multiplier in the production constraint. Optimality implies a common value ξ_t for all firms given by:

$$\xi_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^k + \delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}. \tag{3}$$

Integrating the factor price equations in (2) implies:

$$K_{t-1} = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^k + \delta}{\alpha}\right)^{\alpha-1}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} Y_t \quad \text{and} \quad L_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^k + \delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{-\alpha} Y_t, \tag{4}$$

where Y_t is the total production given by $Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{(\tilde{r}_t^k+\delta)K_{t-1}+\tilde{w}_t L_t}{\xi_t}$. In this setup we still have the following relationship:

$$\frac{K_{t-1}}{L_t} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\tilde{w}_t}{\tilde{r}_t^k+\delta}\right). \tag{5}$$

In the real setup, all firms have $\xi_{j,t} = \xi_t = 1$ and we have the usual definitions of factor prices: $\tilde{r}_t^k + \delta = \alpha Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha-1}$ and $\tilde{w}_t = (1-\alpha) Z_t \left(\frac{K_{t-1}}{L_t}\right)^{\alpha}$.

Firm j sets prices to maximize the present discounted value of profits subject to a price adjustment cost (parameterized by ψ). Denoting the real profits of firm j in period t by $D_{j,t}$, we have that the per-period profits of firm j are given by:

$$D_{j,t} = \frac{P_{j,t}}{P_t}Y_{j,t} - \left(\frac{\tilde{r}_t^k+\delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}\frac{(1-\tau_t^D)}{Z_t}Y_{j,t} - \frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)^2 Y_t - t_t^D,$$

where t_t^D is a lump-sum tax financing the subsidy τ_t^D and $Y_{j,t}$ is the demand for product j by the final good producer. By replacing the results, we obtain:

$$D_{j,t} = \left(\frac{P_{j,t}}{P_t} - \xi_t\left(1-\tau_t^D\right)\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} Y_t - \frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)^2 Y_t - t_t^D. \tag{6}$$

The firm j sets the price schedule $(P_{j,t})_{t\geq 0}$ to maximize the intertemporal profit at date 0:

$$\max_{(P_{j,t})_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \frac{M_t}{M_0} D_{j,t}\right],$$

where we are using β as the discount factor since the households are the owners of the firms and $\frac{M_t}{M_0}$ as the pricing kernel.

The problem of the firm j is then given by:

$$\max_{(P_{j,t})_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t \frac{M_t}{M_0}\left[\left(\left(\frac{P_{j,t}}{P_t}\right)^{1-\epsilon} - \xi_t\left(1-\tau_t^D\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon}\right)Y_t - \frac{\psi}{2}\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)^2 Y_t - t_t^D\right].$$

The first-order condition for the problem above yields:

$$\begin{aligned}
&\left((1-\epsilon)\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} + \epsilon\xi_t\left(1-\tau_t^D\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon}\left(\frac{P_{j,t}}{P_t}\right)^{-1}\right)\frac{Y_t}{P_t} - \psi(\Pi_t-1)\left(\frac{1}{P_{j,t-1}}P_{j,t}\frac{Y_t}{P_{j,t}}\right)\\
&+\beta\mathbb{E}_t\left[\frac{M_{t+1}}{M_t}\psi(\Pi_{t+1}-1)\frac{P_{j,t+1}}{P_{j,t}^2}\frac{Y_{t+1}}{Y_t}Y_t\right] = 0,
\end{aligned}$$

where we defined the gross inflation rate as $\Pi_t = \frac{P_{j,t}}{P_{j,t-1}}$. By manipulating the above equation, we get:

$$\begin{aligned}
&\left((1-\epsilon) + \epsilon\xi_t\left(1-\tau_t^D\right)\left(\frac{P_t}{P_{j,t}}\right)\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon}\frac{1}{P_t}Y_t - \psi(\Pi_t-1)\Pi_t\frac{Y_t}{P_{j,t}}\\
&+\beta\psi\mathbb{E}_t\left[\frac{M_{t+1}}{M_t}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]\frac{Y_t}{P_{j,t}} = 0.
\end{aligned}$$

Now set τ_t^D to be $\frac{1}{\epsilon}$ to obtain an efficient steady state, and since this program gives a solution which is independent of the firm type j, we can define a symmetric equilibrium where $P_{j,t} = P_t$ for all firms j. This means that at the end, we obtain:

$$\left((1-\epsilon)+\epsilon\xi_t\left(\frac{\epsilon-1}{\epsilon}\right)\right)-\psi(\Pi_t-1)\Pi_t+\beta\psi\mathbb{E}_t\left[(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}\right]=0.$$

The above result leads to the Phillips curve in this environment, which will be given by:

$$\Pi_t(\Pi_t-1)=\frac{\epsilon-1}{\psi}(\xi_t-1)+\beta\mathbb{E}_t\left[\Pi_{t+1}(\Pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}\right]. \tag{7}$$

Finally, observe that the real profit of the firm is independent of its type and is given by:

$$D_t=\left(1-\xi_t-\frac{\psi}{2}\left(\Pi_t-1\right)^2\right)Y_t. \tag{8}$$

2.4 Assets

In this environment agents trade two types of assets. The first one is a nominal public debt, whose total supply is denoted by B_t and is determined by the government. This assets pays off a nominal gross and pre-tax interest rate that is predetermined. The nominal before-tax rate between dates $t-1$ and t is known and is given by $\tilde{R}^n_{t-1}$. The associated real interest rate is given by $\frac{\tilde{R}^n_{t-1}}{\Pi_t}$, with Π_t being the gross inflation rate. The other asset is capital-share, which pays the before-tax value of $\tilde{r}^k_t$ as we explained above. The total capital in this economy is given by K_t.

The public debt and capital are assumed to be perfect substitutes and to payoff the same pre-tax interest rate $\tilde{r}_t$. This assumption is akin to the existence of a risk-neutral fund (see Gornemann et al. 2016 among others) holding all interest-bearing assets (i.e., capital and public debt) and selling its shares to agents. The different interest rates are connected by different relationships. First observe that by denoting total assets A_t, we have $A_t = K_t + B_t$. Assuming the existence of a mutual fund that sells shares of this total assets to agents, the non-profit condition of this mutual fund implies:

$$\tilde{r}_tA_{t-1}=\tilde{r}^k_tK_{t-1}+\left(\frac{\tilde{R}^n_{t-1}}{\Pi_t}-1\right)B_{t-1}. \tag{9}$$

Moreover, since capital and public debt are perfect substitutes a non-arbitrage condition is needed. This condition states that one unit of consumption invested in either capital or public bond must entails the same expected return, which means:

$$\mathbb{E}_t\left[\frac{\tilde{R}^n_t}{\Pi_{t+1}}\right]=\mathbb{E}_t[1+\tilde{r}^k_{t+1}]. \tag{10}$$

2.5 Government and its tools

We assume the existence of a benevolent government which has to choose a path stream of public spending denoted by $(G_t)_{t\geq 0}$. This government needs to finance this stream of government spending using several different instruments. First, the government can levy one-period public debt B_t as explained previously. In our environment we assume the existence of an enforcement technology that makes the public debt default-free. Second the government can raise a number of distortionary taxes, which concern labor income and capital revenues. The tax on labor income, denoted by $\mathcal{T}_{i,t}(\tilde{w}_t y_{i,t} l_{i,t})$ for a labor income $\tilde{w}_t y_{i,t} l_{i,t}$, is assumed to be non-linear, and possibly time-varying. We follow Heathcote et al. (2017) and assume that $\mathcal{T}_{i,t}$ is defined as follows:

$$\mathcal{T}_{i,t}(\tilde{w}_t y_{i,t} l_{i,t}) = \tilde{w}_t y_{i,t} l_{i,t} - \kappa_t (\tilde{w}_t y_{i,t} l_{i,t})^{1-\tau_t}, \tag{11}$$

where κ_t captures the level of labor taxation and τ_t the progressivity. Both parameters are assumed to be time-varying and will be planner's instruments. When $\tau_t = 0$, labor tax is linear with rate $1 - \kappa_t$. Oppositely, the case $\tau_t = 1$ corresponds to full income redistribution, where all agents earn the same post-tax income κ_t. The capital tax is linear and denoted by τ_t^k at date t. All of these taxes are proportional taxes and imply a total governmental revenue equal to $\int_i \mathcal{T}_{i,t}(\tilde{w}_t y_{i,t} l_{i,t})\ell(di) + \tau_t^k \tilde{r}_t^k K_{t-1} + \tau_t^k \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1}$.

With these elements, the governmental budget constraint can be written as follows:

$$G_t + \frac{\tilde{R}_{t-1}^n}{\Pi_t} B_{t-1} = D_t + \int_i \mathcal{T}_{i,t}(\tilde{w}_t y_{i,t} l_{i,t})\ell(di) + \tau_t^k \tilde{r}_t^k K_{t-1} + \tau_t^k \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1} + B_t. \tag{12}$$

Using (11) into (12) we have:

$$G_t + \frac{\tilde{R}_{t-1}^n}{\Pi_t} B_{t-1} = D_t + \tilde{w}_t L_t - \kappa_t \int_i (\tilde{w}_t y_{i,t} l_{i,t})^{1-\tau_t}\ell(di) + \tau_t^k \tilde{r}_t^k K_{t-1} + \tau_t^k \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1} + B_t,$$

where $L_t = \int_i y_{i,t} l_{i,t}\ell(di)$ and D_t is given by (8). Notice $D_t = \left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \xi_t Y_t = \left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \left((\tilde{r}_t^k + \delta)K_{t-1} + \tilde{w}_t L_t\right)$, by using the fact that $Y_t \xi_t = (\tilde{r}_t^k + \delta)K_{t-1} + \tilde{w}_t L_t$. Replacing this expression in the above government budget constraint, we obtain:

$$\begin{aligned} G_t + \frac{\tilde{R}_{t-1}^n}{\Pi_t} B_{t-1} = &\left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \left((\tilde{r}_t^k + \delta)K_{t-1} + \tilde{w}_t L_t\right) + \tilde{w}_t L_t \\ &- \kappa_t \int_i (\tilde{w}_t y_{i,t} l_{i,t})^{1-\tau_t}\ell(di) + \tau_t^k \tilde{r}_t^k K_{t-1} + \tau_t^k \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1} + B_t, \end{aligned}$$

We define post-tax rates $r_t, r_t^k, \frac{R_t^n}{\Pi_t}$, and w_t, as follows:

$$r_t = (1 - \tau_t^k)\tilde{r}_t, \quad r_t^k = (1 - \tau_t^k)\tilde{r}_t^k, \quad \frac{R_t^n}{\Pi_t} = (1 - \tau_t^k)\left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right), \quad \text{and} \quad w_t = \kappa_t (\tilde{w}_t)^{1-\tau_t}. \tag{13}$$

Using the above definitions in the government budget constraint we get:

$$G_t + \frac{R_t^n}{\Pi_t} B_{t-1} + B_{t-1} + r_t^k K_{t-1} + w_t \int_i (y_{i,t} l_{i,t})^{1-\tau_t}\ell(di) = \left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \delta K_{t-1} + B_t,$$

Finally using (9) it is possible to show that $(1-\tau_t^k)\tilde{r}_t A_{t-1} = (1-\tau_t^k)\tilde{r}_t^k K_{t-1} + (1-\tau_t^k)\left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1}$ and by using the definition in (13) we finally have $r_t A_{t-1} = r_t^k K_{t-1} + \frac{R_t^n}{\Pi_t} B_{t-1}$ and the final government budget constraint can be written as:

$$G_t + B_{t-1} + r_t A_{t-1} + w_t \int_i (y_{i,t} l_{i,t})^{1-\tau_t}\ell(di) = \left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \delta K_{t-1} + B_t. \tag{14}$$

2.6 Agent's program, resources constraints, and equilibrium definitions

Agents' program. We consider an agent $i \in \mathcal{I}$. Her resources are made of labor income and saving payoffs. The post-tax labor income of an agent with productivity $y_{i,t}$ and supplying the labor effort $l_{i,t}$ amounts to $\tilde{w}_t y_{i,t} l_{i,t} - \mathcal{T}_{i,t}(\tilde{w}_t y_{i,t} l_{i,t}) = w_t (y_{i,t} l_{i,t})^{1-\tau_t}$. Due to the existence of the mutual fund that aggregate all capital and public debt, agents do not make any portfolio choice and we denote by $a_{i,t}$ the holdings of agent i in fund claims at period t. We assume that agents face borrowing constraints and their fund holdings must be such that $a_{i,t} \geq -\overline{a}$. We can also interpret this constraint by reasoning that agent i cannot borrow more than the amount $\overline{a}$. As argued previously these fund claims pay the post-tax interest rate r_t, which means savings payoffs are equal to $(1 + r_t)a_{i,t-1}$ where $a_{i,t-1}$ is the end-of-period-$t-1$ saving of agent i. The agent uses these resources to save and to consume. Since the agent considers the public good path $(G_t)_{t\geq 0}$ as exogenous, her program can formally be expressed as follows:

$$\max_{\{c_{i,t}, l_{i,t}, a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(l_{i,t})\right), \tag{15}$$

$$c_{i,t} + a_{i,t} \leq w_t (y_{i,t} l_{i,t})^{1-\tau_t} + (1 + r_t)a_{i,t-1}, \tag{16}$$

$$a_{i,t} \geq -\overline{a}, c_{i,t} > 0, l_{i,t} > 0, \tag{17}$$

where $\mathbb{E}_0$ is an expectation operator (with respect to aggregate and idiosyncratic risks), and where the initial state $(a_{i,-1}, y_{i,0})$ is given. At date 0, the agent decides her consumption $(c_{i,t})_{t\geq 0}$, her labor supply $(l_{i,t})_{t\geq 0}$, and her saving plans $(a_{i,t})_{t\geq 0}$ that maximize her intertemporal utility of equation (15), subject to a budget constraint (16) and the previous borrowing limit (17). These decisions are functions of the initial state $(a_{i,-1}, y_{i,0})$, the history of idiosyncratic shocks y_i^t, and of the history of shocks z^t.

Assume that in period 0, agent i draw initial asset and productivity $(a_{i,-1}, y_{i,0})$ from an initial distribution $\mathbf{\Lambda}_0(a, y)$ defined over the Borel sets of $[-\overline{a}, \infty) \times \mathcal{Y}$, with $\mathbf{\Lambda}_0(a, y) : [-\overline{a}, \infty) \times \mathcal{Y} \to \mathbb{R}^+$. This allows us to model an economy starting from an arbitrary distribution, including the steady-state distribution.

Remark 1 (Simplifying Notation)**.** *If an agent has an initial state $(a_{i,-1}, y_{i,0})$, and an idiosyncratic history y_i^t at period t, where the aggregate history of shocks is z^t, we will then denote the realization of a state $((a_{i,-1}, y_{i,0}), y_i^t, z^t)$ of any random variable $X_t : ([-\overline{a}, \infty) \times \mathcal{Y}) \times \mathcal{Y}^{t+1} \times \mathbb{R}^{t+1} \to \mathbb{R}$ simply by $X_{i,t}$.*

As a consequence, the aggregation of the variable X_t at period t over all agents will be written as $\int_i X_{i,t} \ell(di)$, instead of the more involved explicit notation where we use the sequential representation and integrate over initial states (of measure Λ) and idiosyncratic histories (of measure θ), considering the realization of the aggregate variable z^t:

$$\sum_{y^t \in \mathcal{Y}^{t+1}} \sum_{y_0 \in \mathcal{Y}} \int_{a_{-1} \in [-\overline{a}, \infty)} \theta_t \left(y^t\right) X_t \left((a_{-1}, y_0), y^t, z^t\right) \mathbf{\Lambda} \left(da_{-1}, y_0\right).$$

All the above means that there exist sequence of functions defined over $([-\overline{a}, \infty) \times \mathcal{Y}) \times \mathcal{Y}^{t+1} \times \mathbb{R}^{t+1}$ and denoted by $(c_t, l_t, a_t)_{t\geq 0}$, such that the agent's optimal decision can be written as:

$$\begin{aligned} c_{i,t} &= c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t), \\ l_{i,t} &= l_t((a_{i,-1}, y_{i,0}), y_i^t, z^t), \\ a_{i,t} &= a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \end{aligned}$$

In what follows we simplify the notation and keep the $i-$ index following the notation of Remark 1.[1]

The first-order conditions (FOCs) associated to the agent's program (15)–(17) can be written as follows:

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t}, \tag{18}$$

$$v'(l_{i,t}) = (1 - \tau_t) w_t y_{i,t} (y_{i,t} l_{i,t})^{-\tau_t} u'(c_{i,t}), \tag{19}$$

where the quantity $\nu_{i,t}$ denotes the discounted Lagrange multiplier on agent i's credit constraint. The Lagrange multiplier $\nu_{i,t}$ will be zero when agent' i is not credit constrained.

Market clearing and resources constraints. The clearing conditions for capital and labor markets can be written as follows:

$$\int_i a_{i,t} \ell(di) = A_t = B_t + K_t \quad \text{and} \quad \int_i y_{i,t} l_{i,t} \ell\,(di) = L_t. \tag{20}$$

For the sake of simplicity, we formulate market clearing by integration over agents i. Equivalently, it could be possible to formulate the market clearing by integrating over initial states and idiosyncratic histories.[2]

The economy-wide resource constraint can be written as:

$$C_t + G_t + K_t = \left(1 - \frac{\psi}{2} \left(\Pi_t - 1 \right)^2 \right) Y_t + K_{t-1} - \delta K_{t-1}, \tag{21}$$

where $C_t = \int_i c_{i,t} \ell(di)$ is the aggregate consumption. To achieve the aforementioned outcome, begin by integrating equation (16) across all agents. This yields $C_t + A_t = w_t \int_i (y_{i,t} l_{i,t})^{1-\tau_t} \ell(di) + (1 + r_t) A_{t-1}$. Subsequently, utilize this derived expression in equation (14), while also considering the relationship $A_t = K_t + B_t$.

Equilibrium definition. The market equilibrium definition can be stated as follows.

Definition 1 (Sequential equilibrium)**.** *A sequential competitive equilibrium is a collection of individual allocations* $(c_{i,t}, l_{i,t}, a_{i,t}, \nu_{i,t})_{t \geq 0, i \in \mathcal{I}}$*, of aggregate quantities* $(K_t, L_t, Y_t, D_t, \xi_t)_{t \geq 0}$*, of price processes* $(r_t, \tilde{r}_t, r_t^k, \tilde{r}_t^k, R_t^n, \tilde{R}_t^n, w_t, \tilde{w}_t)_{t \geq 0}$*, of fiscal policies* $(\tau_t^k, \tau_t, \kappa_t, B_t, G_t)_{t \geq 0}$*, and of monetary policies* $(\Pi_t)_{t \geq 0}$ *such that, for an initial wealth distribution and productivity* $(a_{i,-1}, y_{i,0})_{i \in \mathcal{I}}$*, and*

[1] The existence of those functions can be found in Açikgöz (2016), Cheridito & Sagredo (2016), and Miao (2006).

[2] Using the sequential representation $\int_i a_t^i \ell(di)$ can be written as $\sum_{y^t \in \mathcal{Y}^{t+1}} \sum_{y_0 \in \mathcal{Y}} \int_{a_{-1} \in [-\overline{a}, \infty)} \theta_t\,(y^t)\, a_t\,((a_{-1}, y_0), y^t, z^t)\, \mathbf{\Lambda}\,(da_{-1}, y_0) = A_t(z^t) = A_t$, where we omit the dependence on the history of aggregate states z^t for sake of simplicity.

for initial values of capital stock and public debt verifying $K_{-1} + B_{-1} = \int_i a_{i,-1}\ell(di)$ *and of the initial value of the aggregate shock* z_0*, we have:*

1. *given prices, the functions* $(c_{i,t}, l_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program in equations (15)–(17);*
2. *financial, labor, and goods markets clear at all dates: for any* $t \geq 0$*, equations (20) and (21) hold;*
3. *the government budget is balanced at all dates: equation (14) holds for all* $t \geq 0$*;*
4. *factor prices* $(r_t, \tilde{r}_t, r_t^k, \tilde{r}_t^k, R_t^n, \tilde{R}_t^n, w_t, \tilde{w}_t)_{t\geq 0}$ *are consistent with condition (5), restrictions (9) and (10), and post-tax definitions (13);*
5. *the path for inflation* $(\Pi_t)_{t\geq 0}$ *follows the Phillips curve for all* $t \geq 0$*. In other words, condition (7) holds for all* $t \geq 0$*.*

3 The Ramsey Problem

3.1 The Ramsey program.

The Ramsey program can be expressed in the following fashion using the post-tax notation:

$$\max_{\left(r_t,\tilde{r}_t,r_t^k,\tilde{r}_t^k,R_t^n,\tilde{R}_t^n,w_t,\tilde{w}_t,\tau_t^k,\tau_t,\kappa_t,B_t,G_t,\Pi_t,K_t,L_t,Y_t,D_t,\xi_t,(c_{i,t},l_{i,t},a_{i,t},\nu_{i,t})_{i\in\mathcal{I}}\right)_{t\geq 0}} W_0, \tag{22}$$

$$G_t + B_{t-1} + r_t A_{t-1} + w_t \int_i (y_{i,t} l_{i,t})^{1-\tau_t} \ell(di) = \left(1 - \frac{\psi}{2}(\Pi_t - 1)^2\right) Y_t - \delta K_{t-1} + B_t, \tag{23}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t(y_{i,t} l_{i,t})^{1-\tau_t}, \tag{24}$$

$$a_{i,t} \geq -\bar{a},\ \nu_{i,t}(a_{i,t} + \bar{a}) = 0,\ \nu_{i,t} \geq 0, \tag{25}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1})\Big] + \nu_{i,t}, \tag{26}$$

$$v'(l_{i,t}) = (1 - \tau_t) w_t y_{i,t} (y_{i,t} l_{i,t})^{-\tau_t} u'(c_{i,t}), \tag{27}$$

$$\Pi_t(\Pi_t - 1) = \frac{\epsilon - 1}{\psi}(\xi_t - 1) + \beta \mathbb{E}_t \left[\Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t}\right], \tag{28}$$

$$\int_i a_{i,t} \ell(di) = A_t = K_t + B_t,\ L_t = \int_i y_{i,t} l_{i,t} \ell\,(di)\,, \tag{29}$$

$$\tilde{r}_t A_{t-1} = \tilde{r}_t^k K_{t-1} + \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1\right) B_{t-1}, \tag{30}$$

$$\mathbb{E}_t \left[\frac{\tilde{R}_t^n}{\Pi_{t+1}}\right] = \mathbb{E}_t[1 + \tilde{r}_{t+1}^k], \tag{31}$$

$$w_t = \kappa_t (\tilde{w}_t)^{1-\tau_t}, \tag{32}$$

$$r_t^k = (1 - \tau_t^k) \tilde{r}_t^k, \tag{33}$$

$$\xi_t = \frac{\tilde{r}_t^k + \delta}{\alpha} \frac{K_{t-1}}{Y_t}, \tag{34}$$

$$M_t = \int_i u'(c_{i,t}) \ell(di), \tag{35}$$

$$Y_t = \Big(\int_i a_{i,t-1} \ell(di) - B_{t-1}\Big)^\alpha \Big(\int_i y_{i,t} l_{i,t} \ell\,(di)\Big)^{1-\alpha}. \tag{36}$$

Define $W_0 = \mathbb{E}_0 \left[\sum_{t=0}^\infty \beta^t \int_i \omega_{i,t}(u(c_{i,t}) - v(l_{i,t}))\ell(di)\right]$ as the social welfare function, with $\omega_{i,t}$ being the Pareto weight of agent i in period t. Now, let μ_t be the Lagrange multiplier on the government budget constraint given by (23), γ_t be the Lagrange multiplier on the Phillips curve (28), Γ_t be the Lagrange multiplier on the non-profit condition (30), and Ψ_t be the Lagrange multiplier on the non-arbitrage condition given by (31).

3.2 First-order conditions for the Ramsey program

The Ramsey problem (22)–(36) can be rewritten as:

$$
\begin{aligned}
\mathcal{L} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_{i,t} \left(u\left(c_{i,t}\right) - v\left(l_{i,t}\right)\right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1}\right) u'\left(c_{i,t}\right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,l,t} \left(v'(l_{i,t}) - (1-\tau_t) w_t y_{i,t} (y_{i,t} l_{i,t})^{-\tau_t} u'(c_{i,t})\right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\gamma_t - \gamma_{t-1}\right) \Pi_t \left(\Pi_t - 1\right) Y_t M_t \\
& + \frac{\epsilon - 1}{\psi} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\xi_t - 1\right) Y_t M_t \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(G_t + (1-\delta) B_{t-1} + (r_t + \delta) A_{t-1} + w_t \int_i (y_{i,t} l_{i,t})^{1-\tau_t} \ell(di) - \left(1 - \frac{\psi}{2}\left(\Pi_t - 1\right)^2\right) Y_t - B_t \right) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t \left((1-\tau_t^k) \tilde{r}_t^k K_{t-1} + (1-\tau_t^k) \left(\frac{\tilde{R}_{t-1}^n}{\Pi_t} - 1 \right) B_{t-1} - r_t A_{t-1} \right) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Psi_t \left(\mathbb{E}_t [1 + \tilde{r}_{t+1}^k] - \mathbb{E}_t \left[\frac{\tilde{R}_t^n}{\Pi_{t+1}} \right] \right).
\end{aligned} \tag{37}
$$

The instruments are: $\Pi_t, \tilde{R}_t^n, \tilde{r}_t^k, a_{i,t}, B_t, l_{i,t}, w_t, r_t,$ and τ_t.

FOC with respect to Π_t. Deriving (37) with respect to Π_t yields:

$$
0 = -(\gamma_t - \gamma_{t-1})(2\Pi_t - 1) Y_t M_t - \mu_t \psi (\Pi_t - 1) Y_t - \Gamma_t (1-\tau_t^k) B_{t-1} \frac{-\tilde{R}_{t-1}^n}{\Pi_t^2} + \Psi_{t-1} \beta^{-1} \frac{-\tilde{R}_{t-1}^n}{\Pi_t^2}.
$$

Manipulating the above result we get:

$$
\underbrace{\mu_t \psi (\Pi_t - 1)}_{\text{Total cost due to inflation}} = \underbrace{(\gamma_{t-1} - \gamma_t)(2\Pi_t - 1) M_t}_{\text{Manipulation of real wage with the Phillips curve}} + \underbrace{\left(\Gamma_t (1-\tau_t^k) B_{t-1} - \beta^{-1} \Psi_{t-1}\right) \frac{\tilde{R}_{t-1}^n}{\Pi_t^2 Y_t}}_{\text{Reduction of interest payment on public debt}}.
$$

In the end the final condition is given by:

$$
0 = \mu_t \psi (\Pi_t - 1) + (\gamma_t - \gamma_{t-1})(2\Pi_t - 1) M_t - \left(\Gamma_t (1-\tau_t^k) B_{t-1} - \beta^{-1} \Psi_{t-1}\right) \frac{\tilde{R}_{t-1}^n}{\Pi_t^2 Y_t}. \tag{38}
$$

FOC with respect to $\tilde{R}_t^n$. Deriving (37) with respect to $\tilde{R}_t^n$ yields:

$$
0 = -\beta \Gamma_{t+1} (1 - \tau_{t+1}^k) \frac{B_t}{\Pi_{t+1}} + \Psi_t \frac{1}{\Pi_{t+1}}. \tag{39}
$$

FOC with respect to $\tilde{r}_t^k$. Deriving (37) with respect to $\tilde{r}_t^k$ yields:

$$-\Gamma_t(1-\tau_t^k)K_{t-1} + \left(\frac{\epsilon-1}{\psi}\right)\gamma_t \frac{1}{\alpha} K_{t-1} M_t - \beta^{-1}\Psi_{t-1} = 0.$$

In the end we have:

$$\beta^{-1}\Psi_{t-1} = \left(\left(\frac{\epsilon-1}{\psi}\right)\gamma_t \frac{1}{\alpha} M_t - \Gamma_t(1-\tau_t^k)\right) K_{t-1}. \tag{40}$$

FOC with respect to $a_{i,t}$. Deriving (37) with respect to $a_{i,t}$ yields:

$$\begin{aligned}
0 = {} & \beta^t \int_j \omega_{j,t} u'(c_{j,t}) \frac{\partial c_{j,t}}{\partial a_{i,t}} \ell(dj) \\
& - \beta^t \int_j \left(\lambda_{j,c,t} - (1+r_t)\lambda_{j,c,t-1}\right) u''(c_{j,t}) \frac{\partial c_{j,t}}{\partial a_{i,t}} \ell(dj) \\
& + \beta^t (1-\tau_t) w_t \int_j \lambda_{j,l,t} (y_{j,t})^{1-\tau_t} (l_{j,t})^{-\tau_t} u''(c_{j,t}) \frac{\partial c_{j,t}}{\partial a_{i,t}} \ell(dj) \\
& - \beta^t \left((\gamma_t - \gamma_{t-1}) \Pi_t (\Pi_t - 1) Y_t - \left(\frac{\epsilon-1}{\psi}\right) \gamma_t (\xi_t - 1) Y_t \right) \int_j u''(c_{j,t}) \frac{\partial c_{j,t}}{\partial a_{i,t}} \ell(dj) \\
& + \beta^t \mu_t \left(\left(1 - \frac{\psi}{2}\left(\Pi_t - 1\right)^2\right) \frac{\partial Y_t}{a_{i,t}} - (r_t + \delta) \frac{\partial A_{t-1}}{\partial a_{i,t}} \right) \\
& + \beta^{t+1} \mathbb{E}_t \left[\int_j \omega_{j,t+1} u'(c_{j,t+1}) \frac{\partial c_{j,t+1}}{\partial a_{i,t}} \ell(dj) \right] \\
& - \beta^{t+1} \mathbb{E}_t \left[\int_j \left(\lambda_{j,c,t+1} - (1+r_{t+1})\lambda_{j,c,t}\right) u''(c_{j,t+1}) \frac{\partial c_{j,t+1}}{\partial a_{i,t}} \ell(dj) \right] \\
& + \beta^{t+1} (1-\tau_{t+1}) w_{t+1} \mathbb{E}_t \left[\int_j \lambda_{j,l,t+1} (y_{j,t+1})^{1-\tau_{t+1}} (l_{j,t+1})^{-\tau_{t+1}} u''(c_{j,t+1}) \frac{\partial c_{j,t+1}}{\partial a_{i,t}} \ell(dj) \right] \\
& - \beta^{t+1} \mathbb{E}_t \left[\left((\gamma_{t+1} - \gamma_t) \Pi_{t+1} (\Pi_{t+1} - 1) Y_{t+1} - \left(\frac{\epsilon-1}{\psi}\right) \gamma_{t+1} (\xi_{t+1} - 1) Y_{t+1} \right) \int_j u''(c_{j,t+1}) \frac{\partial c_{j,t+1}}{\partial a_{i,t+1}} \ell(dj) \right] \\
& + \beta^{t+1} \mathbb{E}_t \left[\mu_{t+1} \left(\left(1 - \frac{\psi}{2}\left(\Pi_{t+1} - 1\right)^2\right) \frac{\partial Y_{t+1}}{a_{i,t}} - (r_{t+1} + \delta) \frac{\partial A_t}{\partial a_{i,t}} \right) \right] \\
& - \beta^{t+1} \mathbb{E}_t \left(\left((\gamma_{t+1} - \gamma_t) \Pi_{t+1} (\Pi_{t+1} - 1) - \left(\frac{\epsilon-1}{\psi}\right) \gamma_{t+1} (\xi_{t+1} - 1) \right) M_{t+1} \frac{\partial Y_{t+1}}{\partial a_{i,t}} \right) \\
& + \beta^{t+1} \mathbb{E}_t [\Gamma_{t+1} r_{t+1}] - \beta^{t+1} \mathbb{E}_t \left[\Gamma_{t+1} \left((1 - \tau_{t+1}^k) \tilde{r}_{t+1}^k \right) \right] \\
& + \underbrace{\left(\frac{\epsilon-1}{\psi}\right) \beta^{t+1} \mathbb{E}_t \left[\gamma_{t+1} Y_{t+1} M_{t+1} \frac{\tilde{r}_{t+1}^k + \delta}{\alpha} \frac{(1-\alpha)}{Y_{t+1}} \right]}_{\left(\frac{\epsilon-1}{\psi}\right) \beta^{t+1} \mathbb{E}_t \left[\gamma_{t+1} Y_{t+1} M_{t+1} \xi_{t+1} (1-\alpha) \frac{1}{K_t} \right]}.
\end{aligned}$$

Observe that:

$$\left(\frac{\epsilon-1}{\psi}\right) \beta^{t+1} \mathbb{E}_t \left[\gamma_{t+1} (\xi_{t+1} - 1) M_{t+1} \alpha K_t^{\alpha-1} L_{t+1}^{1-\alpha} \frac{K_t}{K_t} \right] + \left(\frac{\epsilon-1}{\psi}\right) \beta^{t+1} \mathbb{E}_t \left[\gamma_{t+1} Y_{t+1} M_{t+1} \xi_{t+1} (1-\alpha) \frac{1}{K_t} \right] =$$
$$\left(\frac{\epsilon-1}{\psi}\right) \beta^{t+1} \mathbb{E}_t \left[\gamma_{t+1} (\xi_{t+1} - \alpha) M_{t+1} \frac{Y_{t+1}}{K_t} \right].$$

Also, note that we use the fact $K_{t-1} = A_{t-1} - B_{t-1}$ to facilitate taking the First Order Conditions (FOC). The last term of the FOC above arises from observing that: $\xi_t = \left(\frac{\tilde{r}^k_t + \delta}{\alpha}\right) \frac{K_{t-1}}{Y_t}$. Therefore, $\frac{\partial \xi_{t+1}}{\partial a_{i,t}} = (1-\alpha)\frac{\tilde{r}^k_{t+1}+\delta}{\alpha}\frac{1}{Y_{t+1}}$.

Denote:

$$\begin{aligned}\psi_{i,t} = \omega_{i,t} u'(c_{i,t}) - \Bigg[&\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1} - \lambda_{i,l,t}(1-\tau_t) w_t (y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t} \\ &+ \left((\gamma_t - \gamma_{t-1})\Pi_t(\Pi_t - 1) - \left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t - 1)\right) Y_t\Bigg] u''(c_{i,t}).\end{aligned} \tag{41}$$

Using (24) and (36), we obtain:

$$\begin{aligned}\frac{\partial c_{j,t}}{\partial a_{i,t}} &= -1_{i=j}, \\ \frac{\partial A_t}{\partial a_{i,t}} &= 1, \\ \frac{\partial c_{j,t+1}}{\partial a_{i,t}} &= (1+r_{t+1})1_{i=j}, \\ \frac{\partial Y_{t+1}}{\partial a_{i,t}} &= \alpha K_t^{\alpha-1} L_{t+1}^{1-\alpha}.\end{aligned}$$

From which we deduce:

$$\begin{aligned}\psi_{i,t} = \beta \mathbb{E}_t\left[(1+r_{t+1})\psi_{i,t+1}\right] + \beta \mathbb{E}_t &\left[\mu_{t+1}\left(\left(1 - \frac{\psi}{2}\left(\Pi_{t+1}-1\right)^2\right)\underbrace{\alpha K_t^{\alpha-1} L_{t+1}^{1-\alpha}}_{\left(\frac{\tilde{r}^k_{t+1}+\delta}{\xi_{t+1}}\right)} - r_{t+1} - \delta\right)\right] \\ + \left(\frac{\epsilon-1}{\psi}\right)\beta \mathbb{E}_t &\left[\gamma_{t+1}(\xi_{t+1}-\alpha) M_{t+1}\frac{Y_{t+1}}{K_t}\right] - \beta \mathbb{E}_t\left[\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)\alpha \frac{M_{t+1}Y_{t+1}}{K_t}\right] \\ - \beta \mathbb{E}_t &\left[\Gamma_{t+1}(1-\tau^k_{t+1})\tilde{r}^k_{t+1}\right] + \beta \mathbb{E}_t\left[\Gamma_{t+1} r_{t+1}\right].\end{aligned} \tag{42}$$

FOC with respect to B_t. Deriving (37) with respect to B_t yields:

Firstly, it's important to note that we need to replace $K_{t-1} = A_{t-1} - B_{t-1}$ before we can proceed to take the First Order Conditions (FOC).

$$\begin{aligned}0 = \mu_t - \beta \mathbb{E}_t &\left[\mu_{t+1}\left(1 - \delta + \left(1 - \frac{\psi}{2}\left(\Pi_{t+1}-1\right)^2\right)\underbrace{\alpha K_t^{\alpha-1} L_{t+1}^{1-\alpha}}_{\left(\frac{\tilde{r}^k_{t+1}+\delta}{\xi_{t+1}}\right)}\right)\right] \\ - \left(\frac{\epsilon-1}{\psi}\right)\beta \mathbb{E}_t &\left[\gamma_{t+1}(\xi_{t+1}-\alpha) M_{t+1}\frac{Y_{t+1}}{K_t}\right] + \beta \mathbb{E}_t\left[\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)\alpha \frac{M_{t+1}Y_{t+1}}{K_t}\right] \\ - \beta \mathbb{E}_t &\left[\Gamma_{t+1}(1-\tau^k_{t+1})\left(\frac{\tilde{R}^n_t}{\Pi_{t+1}} - 1\right)\right] + \beta \mathbb{E}_t\left[\Gamma_{t+1}(1-\tau^k_{t+1})\tilde{r}^k_{t+1}\right].\end{aligned}$$

In the end we have the following condition:

$$\begin{aligned}\mu_t = \beta\mathbb{E}_t&\left[\mu_{t+1}\left(1-\delta+\left(1-\frac{\psi}{2}\left(\Pi_{t+1}-1\right)^2\right)\left(\frac{\tilde{r}^k_{t+1}+\delta}{\xi_{t+1}}\right)\right)\right] \\ &+\beta\mathbb{E}_t\left[\left(\left(\frac{\epsilon-1}{\psi}\right)\gamma_{t+1}(\xi_{t+1}-\alpha)-\alpha\left(\gamma_{t+1}-\gamma_t\right)\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right)\frac{M_{t+1}Y_{t+1}}{K_t}\right] \\ &+\beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau^k_{t+1})\left(\frac{\tilde{R}^n_t}{\Pi_{t+1}}-1-\tilde{r}^k_{t+1}\right)\right].\end{aligned}\tag{43}$$

By joining (42) and (43) we have:

$$\begin{aligned}\psi_{i,t}-\mu_t = \beta\mathbb{E}_t\left[\psi_{i,t}(1+r_{t+1})\right]-\beta\mathbb{E}_t\left[\mu_{t+1}(1-\delta)\right]+\beta\mathbb{E}_t\left[\mu_{t+1}(-r_{t+1}-\delta)\right]-\beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau^k_{t+1})\tilde{r}^k_{t+1}\right] \\ +\beta\mathbb{E}_t\left[\Gamma_{t+1}r_{t+1}\right]-\beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau^k_{t+1})\left(\frac{\tilde{R}^n_t}{\Pi_{t+1}}-1\right)\right]+\beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau^k_{t+1})\tilde{r}^k_{t+1}\right].\end{aligned}$$

Now denote the following:

$$\hat{\psi}_{i,t}=\psi_{i,t}-\mu_t.\tag{44}$$

Using definition (44) we can easily show:

$$\hat{\psi}_{i,t}=\beta\mathbb{E}_t\left[\hat{\psi}_{i,t}(1+r_{t+1})\right]+\beta\mathbb{E}_t\left[\Gamma_{t+1}\left(r_{t+1}-(1-\tau^k_{t+1})\left(\frac{\tilde{R}^n_t}{\Pi_{t+1}}-1\right)\right)\right].\tag{45}$$

FOC with respect to $l_{i,t}$. Deriving (37) with respect to $l_{i,t}$ yields:

$$\begin{aligned}0 &= \int_j \psi_{j,t}\frac{\partial c_{j,t}}{\partial l_{i,t}}\ell(dj)-\omega_{i,t}v'(l_{i,t})-\lambda_{i,l,t}v''(l_{i,t}) \\ &+\lambda_{i,l,t}(1-\tau_t)(-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t-1}u'(c_{i,t})-\mu_t\Bigg(w_t(1-\tau_t)(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t} \\ &-\left(1-\frac{\psi}{2}\left(\Pi_t-1\right)^2\right)\frac{\partial Y_t}{\partial l_{i,t}}\Bigg)-\left(\gamma_t-\gamma_{t-1}\right)\Pi_t\left(\Pi_t-1\right)M_t\frac{\partial Y_t}{\partial l_{i,t}}+\left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t-1)M_t\frac{\partial Y_t}{\partial l_{i,t}} \\ &+\underbrace{\left(\frac{\epsilon-1}{\psi}\right)\gamma_t Y_t M_t\underbrace{\alpha\frac{\tilde{w}_t}{1-\alpha}\frac{1}{Y_t}y_{i,t}}_{\frac{\partial\xi_t}{\partial l_{i,t}}}}_{\left(\frac{\epsilon-1}{\psi}\right)\gamma_t\alpha M_t\frac{\xi_t Y_t}{L_t}y_{i,t}}.\end{aligned}$$

Notice $\xi_t=\left(\frac{\tilde{w}_t}{1-\alpha}\right)\frac{L_t}{Y_t}$ and $L_t=\int_i y_{i,t}l_{i,t}\ell(di)$. Therefore we have $\frac{\partial\xi_t}{\partial l_{i,t}}=\alpha\frac{\tilde{w}_t}{1-\alpha}\frac{1}{Y_t}y_{i,t}$.

Using (24) and (36) we obtain respectively, $\frac{\partial c_{j,t}}{\partial l_{i,t}}=(1-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t}1_{i=j}$ and

$\frac{\partial Y_t}{\partial l_t^i} = (1-\alpha)K_{t-1}^{\alpha}L_t^{-\alpha}y_{i,t} = F_{L,t}y_{i,t} = \frac{\tilde{w}_t}{\xi_t}y_{i,t}$. So:

$$\begin{aligned}
\omega_{i,t}v'(l_{i,t}) + \lambda_{i,l,t}v''(l_{i,t}) &= (1-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t}\hat{\psi}_{i,t} \\
&+ \lambda_{i,l,t}(1-\tau_t)(-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t-1}u'(c_{i,t}) + \mu_t\left(1-\frac{\psi}{2}\left(\Pi_t-1\right)^2\right)F_{L,t}y_{i,t} \\
&- \left(\gamma_t-\gamma_{t-1}\right)\Pi_t\left(\Pi_t-1\right)M_t\frac{(1-\alpha)Y_ty_{i,t}}{L_t} \\
&+ \underbrace{\left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t-1)M_t\frac{(1-\alpha)Y_ty_{i,t}}{L_t} + \left(\frac{\epsilon-1}{\psi}\right)\gamma_t\xi_tM_t\frac{\alpha Y_ty_{i,t}}{L_t}}_{\left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t-(1-\alpha))M_t\frac{Y_ty_{i,t}}{L_t}}.
\end{aligned}$$

where $\hat{\psi}_t^i = \psi_t^i - \mu_t$. After some manipulation we get:

$$\begin{aligned}
\frac{\omega_{i,t}v'(l_{i,t}) + \lambda_{i,l,t}v''(l_{i,t})}{(1-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t}} &= \hat{\psi}_{i,t} - \lambda_{i,l,t}\tau_t\frac{u'(c_{i,t})}{l_{i,t}} + \mu_t\frac{\left(1-\frac{\psi}{2}\left(\Pi_t-1\right)^2\right)F_{L,t}}{(1-\tau_t)w_t(y_{i,t})^{-\tau_t}(l_{i,t})^{-\tau_t}} \\
+ \left[\left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t-(1-\alpha)) - (1-\alpha)(\gamma_t-\gamma_{t-1})\Pi_t(\Pi_t-1)\right] & \frac{Y_tM_t}{L_t(1-\tau_t)w_t(y_{i,t})^{-\tau_t}(l_{i,t})^{-\tau_t}}.
\end{aligned} \tag{46}$$

FOC with respect to w_t. Deriving (37) with respect to w_t yields:

$$\begin{aligned}
0 = &\int_j \left(\psi_{j,t}\frac{\partial c_{j,t}}{\partial w_t} + \lambda_{j,l,t}(1-\tau_t)(y_{j,t})^{1-\tau_t}(l_{j,t})^{-\tau_t}u'(c_{j,t})\right)\ell(dj) \\
&- \mu_t\int_j (y_{j,t}l_{j,t})^{1-\tau_t}\ell(dj) + \left(\frac{\epsilon-1}{\psi}\right)\gamma_tY_tM_t\frac{\partial \xi_t}{\partial w_t}.
\end{aligned}$$

Using (24) we have: $\frac{\partial c_{j,t}}{\partial w_t} = (y_{j,t}l_{j,t})^{1-\tau_t}$. Since $w_t = \kappa(\tilde{w}_t)^{1-\tau_t}$ we then have $\tilde{w}_t = \left(\frac{w_t}{\kappa}\right)^{\frac{1}{1-\tau_t}}$. This means we can write $\xi_t = \left(\frac{\left(\frac{w_t}{\kappa}\right)^{\frac{1}{1-\tau_t}}}{1-\alpha}\right)\frac{L_t}{Y_t}$. Therefore:

$$\frac{\partial \xi_t}{\partial w_t} = \frac{\frac{1}{1-\tau_t}\left(\frac{w_t}{\kappa}\right)^{\frac{\tau_t}{1-\tau_t}}\frac{1}{\kappa}\frac{L_t}{Y_t}}{1-\alpha}.$$

Replacing this result into the previously obtained First Order Conditions (FOC), we get:

$$\begin{aligned}
0 = &\int_j (y_{j,t}l_{j,t})^{1-\tau_t}\left(\hat{\psi}_{j,t} + \lambda_{j,l,t}(1-\tau_t)u'(c_{j,t})/l_{j,t}\right)\ell(dj) \\
&+ \left(\frac{\epsilon-1}{\psi}\right)\gamma_tM_t\frac{1}{1-\tau_t}\left(\frac{w_t}{\kappa}\right)^{\frac{\tau_t}{1-\tau_t}}\frac{1}{\kappa}\frac{L_t}{1-\alpha}.
\end{aligned} \tag{47}$$

FOC with respect to r_t. Deriving (37) with respect to r_t yields:

$$\begin{aligned} 0 = &\int_j \left(\psi_{j,t} \frac{\partial c_{j,t}}{\partial r_t} + \lambda_{j,c,t-1} u'(c_{j,t}) \right) \ell(dj) \\ &- \mu_t \int_j a_{j,t-1} \ell(dj) + \Gamma_t (B_{t-1} + K_{t-1}). \end{aligned}$$

Now using (24) we have $\frac{\partial c_{j,t}}{\partial r_t} = a_{j,t-1}$. Hence:

$$0 = \int_j \left(\hat{\psi}_{j,t} a_{j,t-1} + \lambda_{j,c,t-1} u'(c_{j,t}) \right) \ell(dj) + \Gamma_t (B_{t-1} + K_{t-1}). \tag{48}$$

FOC with respect to τ_t. Deriving (37) with respect to τ_t yields:

$$\begin{aligned} 0 = &\int_j \psi_{j,t} \frac{\partial c_{j,t}}{\partial \tau_t} \ell(dj) \\ &+ w_t \int_j \lambda_{j,l,t} \frac{\partial}{\partial \tau_t} \left((1-\tau_t)(y_{j,t} l_{j,t})^{1-\tau_t} \right) (u'(c_{j,t})/l_{j,t}) \ell(dj) \\ &- \mu_t w_t \int_j \frac{\partial}{\partial \tau_t} \left((y_{j,t} l_{j,t})^{1-\tau_t} \right) \ell(dj) + \left(\frac{\epsilon - 1}{\psi} \right) \gamma_t Y_t M_t \frac{\partial \xi_t}{\partial \tau_t}. \end{aligned}$$

Using (24) we have $\frac{\partial c_{j,t}}{\partial \tau_t} = w_t \frac{\partial}{\partial \tau_t} \left((y_{j,t} l_{j,t})^{1-\tau_t} \right)$.

Denote $y = (y_{j,t} l_{j,t})^{1-\tau_t}$. Hence, we have $\ln y = (1-\tau_t) \ln(y_{j,t} l_{j,t})$ and $\frac{1}{y} = -\frac{\partial \tau_t}{\partial y} \ln(y_{j,t} l_{j,t})$, which means $\frac{\partial y}{\partial \tau_t} = -\ln(y_{j,t} l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t}$. Then:

$$\frac{\partial}{\partial \tau_t} \left((y_{j,t} l_{j,t})^{1-\tau_t} \right) = -\ln(y_{j,t} l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t}.$$

By the same reasoning now denote $y = \left(\frac{w_t}{\kappa} \right)^{\frac{1}{1-\tau_t}}$. Hence, we have $\ln y = \frac{1}{1-\tau_t} \ln \left(\frac{w_t}{\kappa} \right)$ and $\frac{1}{y} = \frac{\partial \left(\frac{1}{1-\tau_t} \right)}{\partial y} \ln \left(\frac{w_t}{\kappa} \right)$, so $\frac{\partial y}{\partial \tau_t} = \frac{1}{(1-\tau_t)^2} \left(\frac{w_t}{\kappa} \right)^{\frac{1}{1-\tau_t}} \ln \left(\frac{w_t}{\kappa} \right)$. This means:

$$\frac{\partial \xi_t}{\partial \tau_t} = \frac{1}{(1-\tau_t)^2} \left(\frac{w_t}{\kappa} \right)^{\frac{1}{1-\tau_t}} \ln \left(\frac{w_t}{\kappa} \right) \frac{L_t}{(1-\alpha) Y_t}.$$

Observe we then have:

$$\begin{aligned} 0 = &\int_j \hat{\psi}_{j,t} w_t \ln(y_{j,t} l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t} \ell(dj) + w_t \int_j \lambda_{j,l,t} (1-\tau_t) (u'(c_{j,t})/l_{j,t}) \ln(y_{j,t} l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t} \ell(dj) \\ &+ w_t \int_j \lambda_{j,l,t} (u'(c_{j,t})/l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t} \ell(dj) - \left(\frac{\epsilon - 1}{\psi} \right) \gamma_t Y_t M_t \frac{1}{(1-\tau_t)^2} \left(\frac{w_t}{\kappa} \right)^{\frac{1}{1-\tau_t}} \ln \left(\frac{w_t}{\kappa} \right) \frac{L_t}{(1-\alpha) Y_t}. \end{aligned}$$

In the end we have the following expression:

$$
\begin{aligned}
0 = &\int_j (y_{j,t} l_{j,t})^{1-\tau_t} \left(\hat{\psi}_{j,t} + \lambda_{j,l,t} \left(1-\tau_t\right) \left(u'(c_{j,t})/l_{j,t}\right)\right) \ln(y_{j,t} l_{j,t}) \ell(dj) + \\
&\int_j \lambda_{j,l,t} (u'(c_{j,t})/l_{j,t}) (y_{j,t} l_{j,t})^{1-\tau_t} \ell(dj) - \left(\frac{\epsilon - 1}{\psi}\right) \gamma_t M_t \frac{1}{(1-\tau_t)^2} \left(\frac{w_t^{\frac{\tau_t}{1-\tau_t}}}{\kappa^{\frac{1}{1-\tau_t}}}\right) \ln\left(\frac{w_t}{\kappa}\right) \frac{L_t}{(1-\alpha)}.
\end{aligned} \tag{49}
$$

Our identification strategy is to identify Pareto weights ($\omega_{i,t}$) that satisfy the first-order conditions of the social planner (i.e., equations (38) to (49)) and are closest to the utilitarian Pareto weights. This identification method allows us to pinpoint the weights that align with the observed fiscal policies and inequalities within a given economy.

In order to do this, we first assume that the observed fiscal choices in a given economy result from the optimal choices of a benevolent planner, who maximizes intertemporal welfare in a heterogeneous-agent model according to the welfare function:

$$
W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_{i,t} (u(c_{i,t}) - v(l_{i,t})) \ell(di) \right].
$$

The planner is endowed with her own Social Welfare Function and understands distortions and the general-equilibrium effects of all fiscal and monetary instruments. The strategy is then to estimate $\omega_{i,t}$ such that the first-order conditions above are achieved, resulting in the fiscal and inequality outcomes observed in the economy. Moreover, as the number of instruments is finite and there is possibly a vast set of Social Welfare Functions (SWFs) which could rationalize the fiscal outcomes, our second identification assumption is to select among the possible SWFs the one which is closest to the Utilitarian SWF – attributing the same weight to all agents.

Summary of FOCs.

$$0 = \mu_t\psi(\Pi_t - 1) + (\gamma_t - \gamma_{t-1})(2\Pi_t - 1)M_t - \left(\Gamma_t(1-\tau_t^k)B_{t-1} - \beta^{-1}\Psi_{t-1}\right)\frac{\tilde{R}_{t-1}^n}{\Pi_t^2 Y_t}, \tag{50}$$

$$0 = -\beta\Gamma_{t+1}(1-\tau_{t+1}^k)\frac{B_t}{\Pi_{t+1}} + \Psi_t\frac{1}{\Pi_{t+1}}, \tag{51}$$

$$\beta^{-1}\Psi_{t-1} = \left(\left(\frac{\epsilon-1}{\psi}\right)\gamma_t\frac{1}{\alpha}M_t - \Gamma_t(1-\tau_t^k)\right)K_{t-1}, \tag{52}$$

$$\begin{aligned}\psi_{i,t} &= \omega_{i,t}u'(c_{i,t}) - \Big[\lambda_{i,c,t} - (1+r_t)\lambda_{i,c,t-1} - \lambda_{i,l,t}(1-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t} \\ &+ \left((\gamma_t - \gamma_{t-1})\Pi_t(\Pi_t - 1) - \left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t - 1)\right)Y_t\Big]u''(c_{i,t}),\end{aligned} \tag{53}$$

$$\begin{aligned}\psi_{i,t} &= \beta\mathbb{E}_t\left[(1+r_{t+1})\psi_{i,t+1}\right] + \beta\mathbb{E}_t\left[\mu_{t+1}\left(\left(1 - \frac{\psi}{2}(\Pi_{t+1}-1)^2\right)\underbrace{\alpha K_t^{\alpha-1}L_{t+1}^{1-\alpha}}_{\left(\frac{\tilde{r}_{t+1}^k+\delta}{\xi_{t+1}}\right)} - r_{t+1} - \delta\right)\right] \\ &+ \left(\frac{\epsilon-1}{\psi}\right)\beta\mathbb{E}_t\left[\gamma_{t+1}(\xi_{t+1}-\alpha)M_{t+1}\frac{Y_{t+1}}{K_t}\right] - \beta\mathbb{E}_t\left[(\gamma_{t+1}-\gamma_t)\Pi_{t+1}(\Pi_{t+1}-1)\alpha\frac{M_{t+1}Y_{t+1}}{K_t}\right] \\ &- \beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau_{t+1}^k)\tilde{r}_{t+1}^k\right] + \beta\mathbb{E}_t\left[\Gamma_{t+1}r_{t+1}\right],\end{aligned} \tag{54}$$

$$\begin{aligned}\mu_t &= \beta\mathbb{E}_t\left[\mu_{t+1}\left(1 - \delta + \left(1 - \frac{\psi}{2}(\Pi_{t+1}-1)^2\right)\left(\frac{\tilde{r}_{t+1}^k+\delta}{\xi_{t+1}}\right)\right)\right] \\ &+ \beta\mathbb{E}_t\left[\left(\left(\frac{\epsilon-1}{\psi}\right)\gamma_{t+1}(\xi_{t+1}-\alpha) - \alpha(\gamma_{t+1}-\gamma_t)\Pi_{t+1}(\Pi_{t+1}-1)\right)\frac{M_{t+1}Y_{t+1}}{K_t}\right] \\ &+ \beta\mathbb{E}_t\left[\Gamma_{t+1}(1-\tau_{t+1}^k)\left(\frac{\tilde{R}_t^n}{\Pi_{t+1}} - 1 - \tilde{r}_{t+1}^k\right)\right],\end{aligned} \tag{55}$$

$$\hat{\psi}_{i,t} = \psi_{i,t} - \mu_t, \tag{56}$$

$$\hat{\psi}_{i,t} = \beta\mathbb{E}_t\left[\hat{\psi}_{i,t}(1+r_{t+1})\right] + \beta\mathbb{E}_t\left[\Gamma_{t+1}\left(r_{t+1} - (1-\tau_{t+1}^k)\left(\frac{\tilde{R}_t^n}{\Pi_{t+1}} - 1\right)\right)\right], \tag{57}$$

$$\begin{aligned}\frac{\omega_{i,t}v'(l_{i,t}) + \lambda_{i,l,t}v''(l_{i,t})}{(1-\tau_t)w_t(y_{i,t})^{1-\tau_t}(l_{i,t})^{-\tau_t}} &= \hat{\psi}_{i,t} - \lambda_{i,l,t}\tau_t\frac{u'(c_{i,t})}{l_{i,t}} + \mu_t\frac{\left(1 - \frac{\psi}{2}(\Pi_t-1)^2\right)F_{L,t}}{(1-\tau_t)w_t(y_{i,t})^{-\tau_t}(l_{i,t})^{-\tau_t}} \\ &+ \left[\left(\frac{\epsilon-1}{\psi}\right)\gamma_t(\xi_t - (1-\alpha)) - (1-\alpha)(\gamma_t-\gamma_{t-1})\Pi_t(\Pi_t-1)\right]\frac{Y_tM_t}{L_t(1-\tau_t)w_t(y_{i,t})^{-\tau_t}(l_{i,t})^{-\tau_t}},\end{aligned} \tag{58}$$

$$\begin{aligned}0 &= \int_j (y_{j,t}l_{j,t})^{1-\tau_t}\left(\hat{\psi}_{j,t} + \lambda_{j,l,t}(1-\tau_t)u'(c_{j,t})/l_{j,t}\right)\ell(dj) \\ &+ \left(\frac{\epsilon-1}{\psi}\right)\gamma_tM_t\frac{1}{1-\tau_t}\left(\frac{w_t}{\kappa}\right)^{\frac{\tau_t}{1-\tau_t}}\frac{1}{\kappa}\frac{L_t}{1-\alpha},\end{aligned} \tag{59}$$

$$0 = \int_j\left(\hat{\psi}_{j,t}a_{j,t-1} + \lambda_{j,c,t-1}u'(c_{j,t})\right)\ell(dj) + \Gamma_t(B_{t-1} + K_{t-1}), \tag{60}$$

$$\begin{aligned}0 &= \int_j (y_{j,t}l_{j,t})^{1-\tau_t}\left(\hat{\psi}_{j,t} + \lambda_{j,l,t}(1-\tau_t)(u'(c_{j,t})/l_{j,t})\right)\ln(y_{j,t}l_{j,t})\ell(dj) + \\ &\int_j \lambda_{j,l,t}(u'(c_{j,t})/l_{j,t})(y_{j,t}l_{j,t})^{1-\tau_t}\ell(dj) - \left(\frac{\epsilon-1}{\psi}\right)\gamma_tM_t\frac{1}{(1-\tau_t)^2}\left(\frac{w_t^{\frac{\tau_t}{1-\tau_t}}}{\kappa^{\frac{1}{1-\tau_t}}}\right)\ln\left(\frac{w_t}{\kappa}\right)\frac{L_t}{(1-\alpha)}.\end{aligned} \tag{61}$$

4 Quantitative Investigation

In this paper, our objectives are twofold: firstly, to construct calibrated models reflecting the economic conditions of the United Kingdom during two distinct periods: 2007 and 2021. We chose 2007 as our initial point of interest as it predates both the financial crisis and the COVID-19 pandemic, both of which significantly reshaped fiscal structures. Conversely, we selected 2021 to capture the post-COVID-19 landscape. We posit that the emergence of prolonged periods of high debt may signal a new norm, indicating a potential shift to a different steady state.

Focusing on the dynamics of real variables, public debt, and inflation post-shocks, rather than the optimality of the tax system, we initially calibrate parameters to establish a realistic steady state based on the fiscal policies and observed inequalities of the UK in 2007 and 2021. Subsequently, we draw upon the literature on the inverse taxation problem to empirically estimate the Social Welfare Function. This ensures that the observed fiscal policy aligns optimally with the social planner's perspective in the steady state.

Through this calibration approach, we observe the dynamics of the fiscal system by initializing with a realistic allocation at period-0. By employing this strategy and introducing various shocks to the economy, we analyze how fiscal instruments respond, along with the behavior of real variables, public debt, inflation, and capital dynamics. As these shocks are transitory, we verify that the fiscal policy parameters eventually return to the initial steady state, assumed to be optimal. Continuing our exploration, we delve into an analysis of how inflation responds when fiscal policy instruments are absent, aiming to discern potential disparities between the UK's circumstances in 2021 and those of 2007.

Next, we outline the calibration process for the UK economy in both 2007 and 2021.

4.1 The UK economy in 2007

The estimation parameters are gathered in Table 2, and we detail below our calibration strategy.

Preference parameters. The period is a quarter. The discount factor is set to $\beta = 0.99$ so as to match a realistic capital-to-output ratio (K/Y). The period utility of equation (1) is specified such that $u(c_{i,t}) = \frac{c_{i,t}^{1-\gamma}-1}{1-\gamma}$ with $\gamma = 2$ and $v(l_{i,t}) = \frac{1}{\chi}\frac{l_{i,t}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$. Furthermore, the Frisch elasticity for labor is set to $\varphi = 0.5$, which is recommended by Chetty et al. (2011) for the intensive margin. We set the labor-scaling parameter to $\chi = 0.058$, which implies normalizing the aggregate labor supply to 0.39.

Technology and TFP shock. The production function is of the Cobb-Douglas form and such that $Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$. The capital share is set to the standard value, $\alpha = 36\%$, while the depreciation rate is set to $\delta = 2.5\%$. The productivity shock process is a standard AR(1) process with $Z_t = Z_0 e^{z_t}$ and $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$, where $\varepsilon_{z,t} \sim_{\text{IID}} \mathcal{N}(0, \sigma_z^2)$. In the exercises below, we examine various parameters related to the Total Factor Productivity (TFP) shock.

Idiosyncratic labor risk and taxes. Various estimations of the idiosyncratic process can be found in the literature. As argued previously, the productivity follows an AR(1) process: $\log y_t = \rho_y \log y_t + \varepsilon_{y,t}$, with $\varepsilon_{y,t} \sim_{\text{IID}} \mathcal{N}(0, \sigma_y^2)$. In this paper, our identification strategy relies on employing the two parameters identifying this process, namely (ρ_y, σ_y) to pinpoint key moments for the UK economy in 2007. We also incorporate parameters τ and κ, which are linked to the progressivity of the labor tax, to aid in identifying these key moments. The primary target moment is the debt-to-GDP (B/Y) ratio, crucial for replicating a realistic financial market equilibrium. Utilizing data from the Office for National Statistics (ONS) for the UK economy, the debt-to-GDP ratio was 43.1% in 2007. The second target pertains to the Gini coefficient of income post taxes and transfers, for which we also rely in the ONS data to capture income inequality. The targeted value for the UK economy in 2007 is 0.386. Finally, the third moment concerns the variance of income, which correlates with the Gini coefficient of original income before taxes and benefits. The targeted moment for this variable is 53.5 for the UK economy in 2007, where we once again rely on data obtained from the Office for National Statistics (ONS).

By employing the Simulated Method of Moments, we estimate the aforementioned parameters to reproduce the data targets. The calibration features an autocorrelation $\rho_y = 0.995$ and a standard deviation $\sigma_y = 0.07$. The estimated parameters governing the progressivity of the labor taxes are estimated to be such that $\tau_t = 0.05$ and $\kappa_t = 0.62$. Finally, the capital tax for the UK economy in 2007 is set to be $\tau_t^k = 38\%$, using estimations obtained from the Bank of England. The AR(1) process is discretized using Rouwenhorst (1995) procedure, with 7 states. Using those calibrated parameters we obtain a Gini index for pre-tax income equal to 0.53, which is very close to the target value of 0.535 reported in Table 1. Moreover, it implies a Gini index of post-tax and transfers of 0.384, which is also close to the one reported in Table 1. Finally the model generate a debt-to-GDP ratio of 42.5 %, similar to the target value of 43.1 % reported in Table 1. The model also implies a public-spending-to-GDP ratio equal to 27.6% and tax revenues amount to 29.3% of GDP.

In addition, the model predicts a consumption-to-GDP ratio of 46.8% and a variance of consumption equal to 29.2%. The Gini of wealth generated by the model is given by 0.689. We gather the model implications in Table 3. These implications show that our tax system provides a good approximation of the re-distributive effects of the actual tax system and that our model is able to replicate key statistics variables for the UK economy. This confirms the results of Heathcote et al. (2017) and Dyrda & Pedroni (2018).

4.2 The UK economy in 2021

The estimated economy of United Kingdom in 2021 has a lot of similarities with the UK in 2007. For the sake of simplicity we use the same period and the same functional forms. Below we go over the details. The calibration parameters can be found, as those for the UK economy in 2007 in Table 2.

		UK 2007		UK 2021	
Parameter	Description	Value	Target or ref.	Value	Target or ref.
Preference parameters					
β	discount factor	0.99	$K/Y = 2.56$	0.99	$K/Y = 2.56$
u	utility function	$\cdot$	$\gamma = 2.0$	$\cdot$	$\gamma = 2.0$
φ	Frish elasticity	0.5	Chetty et al. (2011)	0.5	Chetty et al. (2011)
χ	hours worked	0.39	Own calculations	0.35	Own calculations
α	capital share	36%	Profit Share, ONS	36%	Profit Share, ONS
δ	depreciation rate	2.5%	Standard value	2.5%	Standard value
Fiscal policy parameters					
τ_t^k	capital tax	0.38	Data BoE	0.35	Data BoE
τ_t	progressivity labor tax	0.05	Table 1	0.22	Table 1
κ_t	scaling labor tax	0.62	Table 1	0.775	Table 1
Productivity parameters					
σ^y	std. err. productivity	0.07	Table 1	0.09	Table 1
ρ^y	autocorr. productivity	0.995	Table 1	0.992	Table 1

Table 2: Parameter values.

Preference parameters. The period is a quarter. The discount factor is set to $\beta = 0.99$. The Frisch elasticity for labor is set to $\varphi = 0.5$. Fixing the labor-scaling parameter to $\chi = 0.053$ means we normalize the aggregate labor supply to 0.35.

Technology and TFP shock. The production function follows the Cobb-Douglas form: $Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$. Here, the capital share is set to the standard value of $\alpha = 36\%$, and the depreciation rate is $\delta = 2.5\%$. The productivity shock process follows a standard AR(1) process: $Z_t = Z_0 e^{z_t}$, where $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$, and $\varepsilon_{z,t} \sim_{\text{IID}} \mathcal{N}(0, \sigma_z^2)$.

Idiosyncratic labor risk and taxes. The productivity follows an AR(1) process: $\log y_t = \rho_y \log y_t + \varepsilon_{y,t}$, with $\varepsilon_{y,t} \sim_{\text{IID}} \mathcal{N}(0, \sigma_y^2)$. As before we use (ρ_y, σ_y) to pinpoint key moments for the UK economy in 2021. We also incorporate labor parameters τ and κ to aid in identifying these key moments. The primary target moment is the debt-to-GDP (B/Y) ratio, which according to Table 1 was 101% in 2021. The second target is the Gini coefficient of income post taxes and transfers, which is 0.299. Finally, the third moment concerns the variance of income, which correlates with the Gini coefficient of original income before taxes and benefits. The targeted moment for this variable is 50.2 for the UK economy in 2021, where we once again rely on data obtained from the Office for National Statistics (ONS).

By employing the Simulated Method of Moments, we estimate the aforementioned parameters to reproduce the data targets. The calibration features an autocorrelation $\rho_y = 0.992$ and a standard deviation $\sigma_y = 0.09$. The estimated parameters governing the progressivity of the labor taxes are estimated to be such that $\tau = 0.22$ and $\kappa = 0.775$. Finally, the capital tax for the UK economy in 2007 is set to be $\tau^k = 35\%$, using estimations obtained from the Bank of England. The AR(1) process is discretized using Rouwenhorst (1995) procedure, with 7 states.

Using those calibrated parameters we obtain a Gini index for pre-tax income equal to 0.50, which is very close to the target value of 0.502 reported in Table 1. Moreover, it implies a Gini index of post-tax and transfers of 0.357, which is close to the one reported in Table 1. Finally the model generate a debt-to-GDP ratio of 112 %. The model also implies a public-spending-to-GDP ratio equal to 14.6% and tax revenues amount to 19.2% of GDP.

In addition, the model predicts a consumption-to-GDP ratio of 59.8% and a variance of consumption equal to 19%. The Gini of wealth generated by the model is given by 0.633. We gather the model implications in Table 3.

	UK 2007		UK 2021	
	Model	Data	Model	Data
B/Y	42.5%	43%	112%	101%
Gini for pre-tax income	53%	53.5%	50.0%	50.2%
Gini for post-tax income	38.4%	38.6%	35.7%	29.9%

Table 3: Model implications for key variables. Empirical values are discussed in Sections 4.1 and 4.2. Data values are summarized in Table 1.

4.3 Numerical tools and Estimation of Pareto Weights

As explained above, our estimation procedure identifies Pareto weights such that the first order conditions of the social planner are satisfied (i.e., equations (38) to (49)) and which are the closest to the utilitarian Pareto weights. The methodology and explanations here are drawn from Le Grand et al. 2022. Below we go over a brief detail of the numerical tool we use to solve the Ramsey problem outlined by equations (22)–(36).

The main issue to identify those weights arise because the Ramsey problem of Section 3.1 involves a joint distribution across wealth and Lagrange multipliers, which leads to a high-dimensional object with a high number of difficulties for the program resolution, especially in the presence of aggregate shocks. For instance, this joint distribution affects the planner's instruments in a non-obvious way, which makes the methods based on perturbation of a well-identified steady-state not usable for solving such problems (as Reiter 2009, Boppart et al. 2018, Bayer et al. 2019 or Auclert et al. 2019).

Due to it, in order to identify the Pareto weights we use the Lagrangian approach method developed in LeGrand & Ragot (2022). Basically this method allows us to compute the steady-state allocation and derive a finite number of equations that can simulate by perturbation the dynamics of the Ramsey program for small aggregate shocks. The idea is to build an aggregation of the Bewley model (thus for a given policy and no aggregate shock) in which agents with the same history over last N (where N is a fixed horizon) periods are aggregated into an unique "agent". This method implies that the "aggregate" agent is endowed with the average wealth and average allocation of all individuals with this N-period history.

To understand this method take an agent i who has at period-t the history $\{y_{i,0}, ..., y_{i,t}\}$. Let $N \geq 0$ be a truncation length. The key step of the aggregation consists in assigning to all agents sharing the same idiosyncratic history over the last $N \geq 0$ periods the same wealth and the same allocation. Such a N-period history will be said to be a truncated history and for a history $y^t = \{y_0, \dots, y_{t-N}, y_{t-N+1}, \dots, y_{t-1}, y_t\}$, this corresponds to the N-length vector denoted $y^N = \{y^N_{t-N+1}, \dots, y^N_{t-1}, y^N_t\}$. To sum up we can represent the truncated history of an agent i whose idiosyncratic history is y^t as:

$$y^t = \{\underbrace{y_0, \dots, y_{t-N-2}, y_{t-N-1}, y_{t-N}}_{\xi_{y^N}}, \underbrace{y^N_{t-N+1}, \dots, y^N_{t-1}, y^N_t}_{=y^N}\},$$

where the parameter ξ_{y^N} captures the residual heterogeneity for the truncated history y^N and is built such that the truncated model will be an exact aggregation of the underlying Bewley model in the absence of aggregate shocks. To clarify this take the history $y^N = \{y^N_{t-N+1}, ..., y^N_{t-1}, y^N_t\}$ and take an agent i such that $\{y_{i,t-n+1}, ..., y_{i,t-1}, y_{i,t}\} = \{y^N_{t-N+1}, ..., y^N_{t-1}, y^N_t\}$. The truncation method then consists in aggregate all agents with the same truncated history, in other words we take all agents i with the same history y^N and then express the model in terms of these groups of agents. This aggregation procedure generate the so-called truncated model, which is a finite-space problem. In the truncated model, the "agent" who is the aggregation of agents with the same history is assumed to have full risk-sharing within each truncated history and thus "forgets" the heterogeneity in histories before the aggregation as discussed in LeGrand & Ragot 2023. The heterogeneity is captured by the parameter ξ_{y^N} and by using this parameter we can go from the truncated model to the original Bewley model.[3]

In Appendix we present the truncated model of the Section 3.1, as well as the FOC for the planner in this case. Finally we write the FOC at the steady state and using simple matrix algebra we pin down the weights.[4] We then show that the Pareto weights satisfy the FOC of the planner.

In Le Grand et al. 2022 they consider a truncation length of $N = 5$, although the main characteristic of the results does not change when we consider a longer truncation length.[5] In this method if you select for instance 10 idiosyncratic productivity levels, this implies $10^5 = 100000$ different truncated histories. The Pareto weights are estimated such that histories with the same productivity level in the beginning of the truncation will be assigned the same weight (i.e., if $y^N_t = \tilde{y}^N_t$ such that $y^N_t \in y^N$ and $\tilde{y}^N_t \in \tilde{y}^N$ with $y^N \neq \tilde{y}^N$ then $\omega(y^N) = \omega(\tilde{y}^N)$). In the end this means we will have a set of 10 Pareto weights, one for each possible value that the idiosyncratic variable can assume.

The method as the one highlighted above is easy to implement but the problem is that it considers many histories that are unlikely to be experienced by the agents. By the law of large numbers those histories concern a very small set of agents. Because of this issue we opt to use the method developed in Le Grand & Ragot 2022*b*, where they propose to use a refined truncation method. The idea is that histories which are more likely to occur can be replaced by a set of histories with higher truncation lengths. Take the truncated history (y_1, y_1) $(N = 2)$. They show that this history can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have

[3] Le Grand & Ragot 2018 and Le Grand et al. 2022 present this methodology in a greater detail.

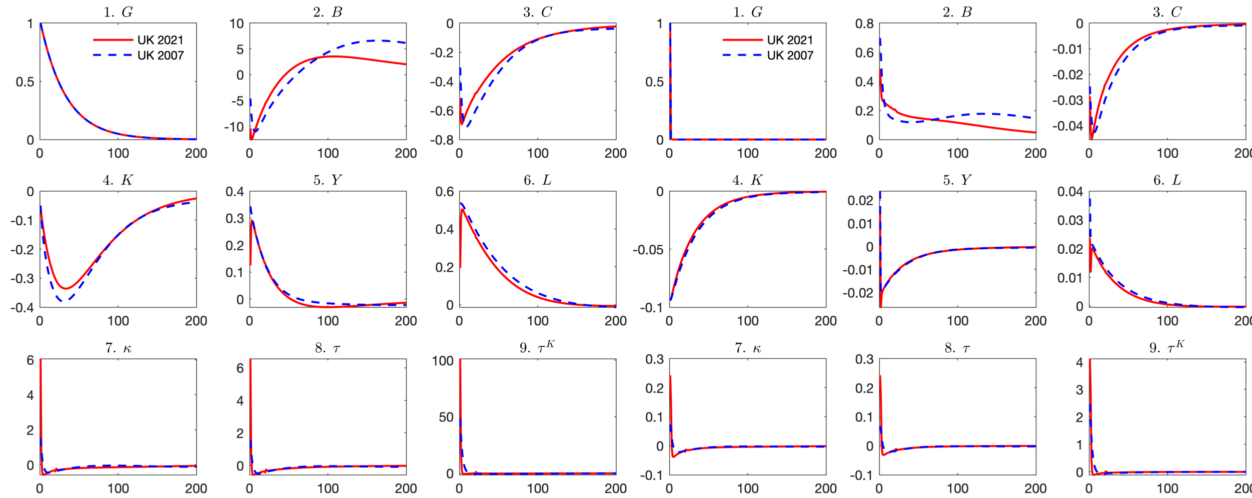
[4] For more details of this approach see Le Grand et al. 2022, where they show the truncated model for a similar problem but without the presence of Monetary policy through a specification of a well defined Social Welfare Function and also using a parametric functional form for the weights in a fashion similar to Heathcote & Tsujiyama (2021).

[5] Le Grand & Ragot 2022*a* showed that the truncated allocation converges to the true one when the truncation length increases. The question is then quantitative, and Ragot & Legrand 2023 showed that a tractable truncation length provides accurate results.

been in productivity y_1 for two consecutive periods is into $Card(\mathcal{Y})$ truncated histories. By doing this the number of histories will be a linear function of the maximum truncation length, instead of an exponential function as explained above.

4.4 Model Dynamics with Optimal Fiscal and Monetary Policy

We begin by simulating the economy post-shocks, employing a solution to the Ramsey problem (22)–(36). Initially, we analyze the trajectory of fiscal variables $(\tau_t^k, \tau_t, \kappa_t, B_t)$ following various shocks occurring at time $t = 0$. Specifically, we first focus on scenarios where the social planner lacks access to monetary policy tools, with fiscal policy instruments being the sole choice variables.



(a) High persistence (b) Low persistence

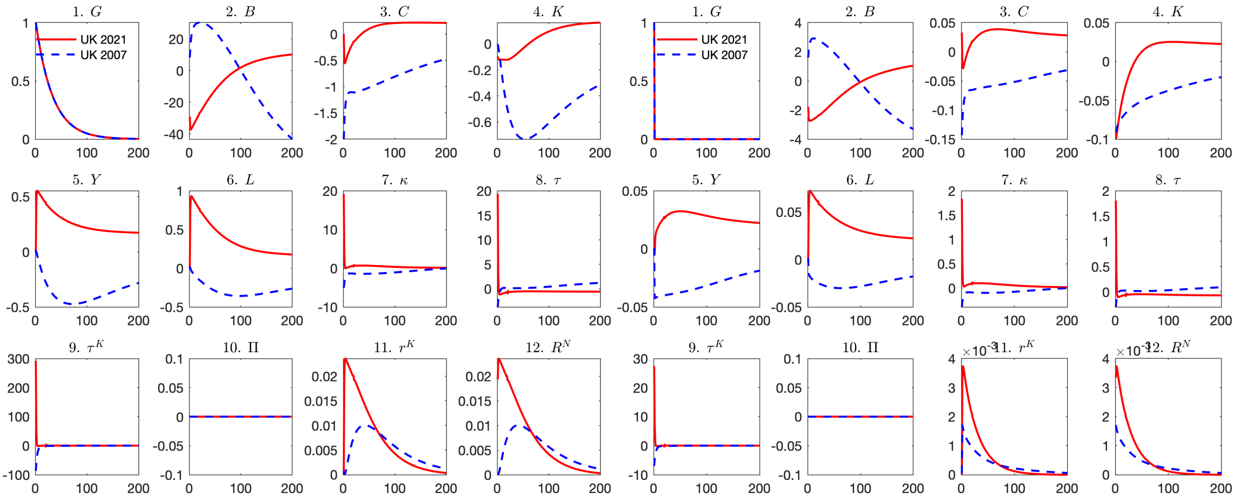
Figure 2: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a positive spending shock with varying levels of persistence.

Figure 2 plots the dynamics of public debt B, consumption C, capital K, output Y, labor L, scaling labor tax κ, progressivity of labor tax τ, and capital tax τ^k in proportional deviations, for two levels of persistence. Panel (a) depicts the outcomes for the high-persistence case ($\rho_z = 0.97$), while panel (b) showcases the results for the low-persistence scenario ($\rho_z = 0.01$). The shock here is assumed to reach the government expenses G_t and has the same structure as the TFP shock outlined above (i.e., $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$).

Observing both panels, we note that the capital in the UK economy experiences a decline after the positive spending shock, indicating a reduction in private savings necessary for implementing consumption smoothing. A comparison between the two scenarios reveals distinct optimal strategies for the social planner. In the low-persistence case, the planner opts to increase public debt as a means of providing a store of value to counteract the fall in capital. Conversely, in the high-persistence scenario, the sustained decline in capital makes the strategy of increasing public debt more costly, as it would necessitate raising taxes in periods of low capital, thus hindering future debt stabilization efforts. Consequently, the planner refrains from increasing public debt, anticipating the need for substantial tax hikes in the future to manage debt levels effectively.

Notably, comparing the situations in 2007 and 2021, we observe that the increase in taxes required to stabilize public debt is higher in 2021 than in 2007 across both scenarios. This discrepancy is attributed to the higher public debt levels in 2021, necessitating more substantial mechanisms for debt stabilization.

We now shift our focus to the scenario where we examine the evolution of fiscal variables $(\tau_t^k, \tau_t, \kappa_t, B_t)$ alongside monetary variables $(\Pi_t, \tilde{R}_t^n)$, representing the solution to the problem outlined by equations (22)–(36). The outcomes of this analysis are presented in Figure 3.



(a) High persistence (b) Low persistence

Figure 3: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a positive spending shock with varying levels of persistence and Optimal Monetary Policy.

Upon examining Figure 3, we observe a scenario where the planner can utilize both fiscal and monetary instruments. Notably, the responses are comparable regardless of whether the shock exhibits high or low persistence. In the case of the UK in 2021, following a positive spending shock, capital diminishes due to increased interest rates. While the planner aims to bolster consumption by increasing public debt, this avenue is restricted given the already elevated debt-to-GDP ratio. Further amplifying public debt would necessitate tax hikes during periods of low capital, prompting the immediate imposition of tax increases to stabilize public debt.

In contrast, the fiscal policy of 2007 appears to be counter-cyclical. Despite the reduction in capital induced by increased government spending, public debt escalates, indicating an immediate need for government borrowing to finance expenditures. This rise in public debt in 2007 can be partly attributed to the combined effect of the substantial decline in capital, particularly pronounced during that period, and the wealth effect generated by government spending, which results in a reduction in labor supply. Both effects contribute to a recession, prompting tax cuts aimed at stimulating labor supply, capital accumulation, and consequently economic growth. Although lower taxes encourage consumption, they also contribute to a larger budget deficit and subsequently higher public debt, especially as government spending remains elevated initially. This contrasting fiscal response between 2007 and 2021 underscores the severity of the capital decline and the wealth effect affecting labor supply. In both scenarios, the planner maintains a path of stable inflation, utilizing fiscal instruments to mitigate shocks.

Comparing both scenarios reveals contrasting dynamics in the optimal trajectory of public debt, particularly concerning different levels of shock persistence and the presence of monetary policy tools. In the absence of monetary instruments, the optimal response to a positive spending shock varies depending on shock persistence.

The introduction of monetary policy tools alters the response dynamics of public debt to shocks. Under this framework, the planner may choose to increase public debt even when the shock persists, recognizing that the reduction in capital could be more pronounced and labor supply could be reduced due to a wealth effect. This strategic decision arises from the necessity, in this economic context, of significantly raising interest rates to maintain stable inflation, thereby amplifying the impact on capital. Consequently, increasing public debt emerges as a proactive measure to mitigate the adverse effects of increased government expenses. This strategic approach reflects the essence of the UK economy's response in 2007.

4.5 Model Dynamics with defined Fiscal and Monetary rules

We posit that if the response in terms of fiscal policy parameters is more pronounced for the UK economy in 2021 compared to 2007, the absence of fiscal tools might prompt the use of monetary policy, potentially deviating from the assumed optimal zero inflation path. Taxpayers may resist tax hikes but may have less control over inflation, making it a more feasible option. Moreover, if optimal shock adjustment through taxes is unfeasible, introducing inflation could spread adjustment costs more broadly across society.

Inflation effectively functions as a hidden tax on money holdings by allowing prices to adjust, distributing adjustment costs evenly. Unlike tax hikes, which can disproportionately burden certain groups and face implementation challenges, inflation offers policymakers flexibility during economic stress. It allows them to reduce debt burden without resorting to unpopular tax hikes or spending cuts, preserving fiscal flexibility. It can also stimulate economic activity by reducing real debt value, encouraging spending and investment, especially during economic downturns when traditional tools are limited.

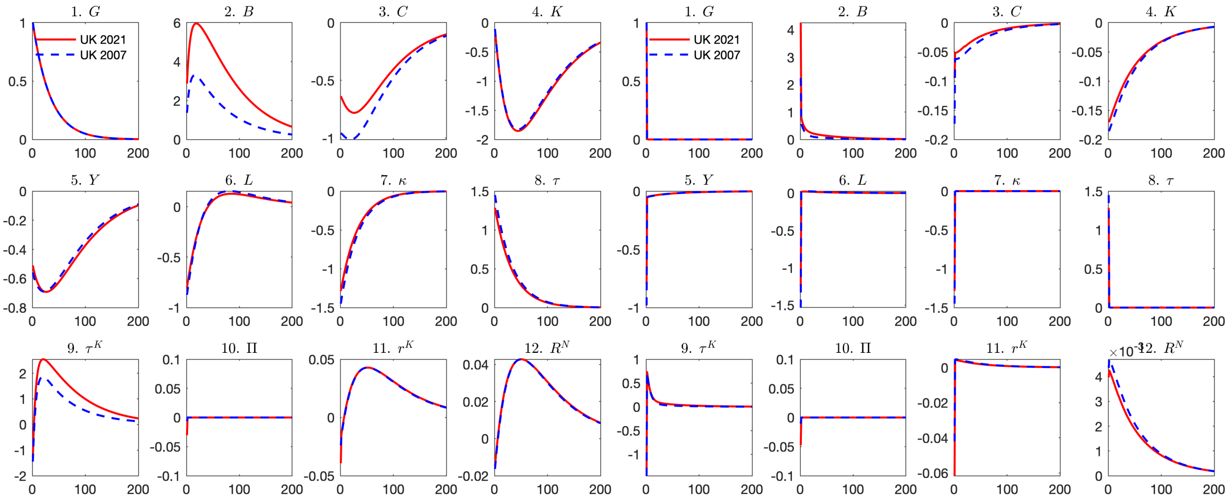
While using inflation as a policy tool has potential adverse consequences and remains controversial, it can effectively manage economic shocks. In the subsequent analysis, we explore the optimality conditions of fiscal policy instruments in the Ramsey problem (22)–(36) and the possibility of adjustment mechanisms through inflation.

4.5.1 Model Dynamics with Fiscal rules

First we consider fiscal rules of the following type:

$$\Xi_t = \Xi + \phi_{\Xi}(B_{t-1} - B), \tag{62}$$

where $\Xi_t \in \{\kappa_t, \tau_t^k, \tau_t\}$. In this case we solve the problem given by (22)–(36), but instead of considering that the taxes solve the optimal problem of the Social Planner and follows the FOC given by equations (47), (48), and (49), we assume they simply follow the rule above and adjust in such a way that they eventually come back to the steady state.



(a) High persistence (b) Low persistence

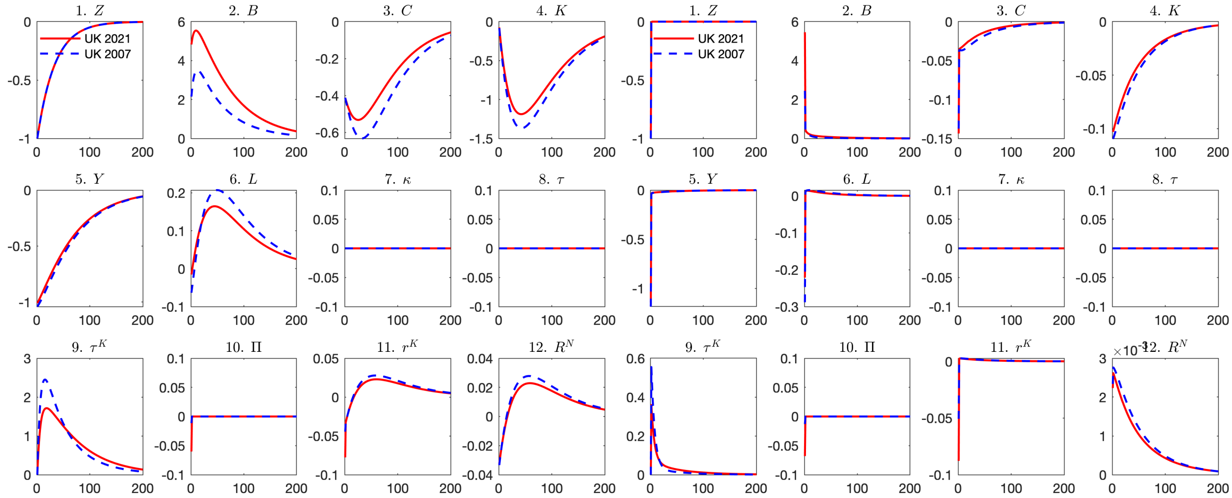
Figure 4: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a positive spending shock with varying levels of persistence and defined fiscal rules.

Upon examining Panels (a) of Figures 3 and 4 for the year 2021, it becomes apparent that restricting tax adjustments to lower values exacerbates the decline in capital. In contrast to the previous scenario, where the UK economy in 2021 exhibited a less severe decline in capital due to higher fiscal policy parameter adjustments, the current observation suggests a heightened need for economic adjustment mechanisms. Moreover, unlike the previous case, the fact that taxes do not adjust generates a wealth effect, which reduces labor supply and triggers a recession. Notably, the increase in government spending necessitates a rise in the public deficit to maintain budget equilibrium, since taxes do not adjust as before.

In the 2021 scenario, to stimulate the economy, the social planner opts to reduce interest rates. However, the reduction in interest rates alleviate debt servicing costs leading to additional borrowing, exacerbating debt levels, as we can see by the increase in public debt reaching its peak after a couple of periods. To counteract this surge in public debt, the social planner opts to reduce inflation, thereby increasing the real cost of debt and facilitating a return to its steady-state value. This strategic maneuver results in an optimal inflation path that deviates from the previously analyzed zero inflation scenario, reflecting the dynamic nature of economic adjustments. Notably, government spending necessitates a rise in public debt to maintain budget equilibrium, a response more pronounced than that observed in 2007.

The primary disparity between Figures 3 and 4 arises in the year 2021, where fiscal rules were previously allowed to adjust optimally, resulting in a substantial increase. Notably, the shock's persistence does not influence the trajectory of variable evolution post-shock under this scenario. This ultimately highlights the importance of fiscal tools as an adjustment mechanism, as the small adjustment in response to increased government spending generates a wealth effect that impacts the labor supply, leading to different results even in the real economy.

Transitioning to the analysis of negative productivity shocks, we now delve into elucidating their implications.



(a) High persistence (b) Low persistence

Figure 5: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a negative productivity shock with varying levels of persistence and defined fiscal rules.

The effects observed when comparing Figures 5 and 4 are quite similar, with the main distinction being the presence of a negative productivity shock in the latter. This shock results in a more pronounced reduction in output, attributable to a sharper decline in capital compared to the scenario analyzed in Figure 4.

The diminished output coupled with the capital decline prompts the social planner to prioritize increasing public debt, the primary available fiscal tool for mitigating the shock while maintaining budget equilibrium. Notably, the increase in public debt is more pronounced in 2021 compared to 2007, indicative of the heightened need for fiscal policy adjustment mechanisms in 2021 relative to 2007.

The subsequent rise in public debt, coupled with the decrease in interest rates, encourages increased debt acquisition by agents, resulting in a peak in the initial periods. To counteract this trend and elevate the cost of acquiring public debt, the social planner adopts debt deflation measures to raise the cost of debt, ultimately aligning its value with the steady state. In both scenarios, the optimal path for inflation entails a reduction, signifying a departure from the previous assumption of zero inflation as the optimal strategy. Although the adjustment is not substantial, it underscores the evolving nature of optimal inflation policy.

Comparing the results obtained from the simulations sheds light on the role of inflation as an adjustment mechanism in the absence of optimality decisions regarding fiscal tools. Initially, when analyzing the trajectory of fiscal variables following various shocks without access to monetary policy tools, we observed distinct optimal strategies for the social planner depending on shock persistence. However, transitioning to scenarios where monetary policy instruments are introduced alters the response dynamics of public debt to shocks. Here, the planner may opt to increase public debt even in the face of persistent shocks. This strategic shift reflects the evolving nature of economic adjustments and underscores the role of inflation in stabilizing the economy.

The comparison between Figures 3 and 4 for the year 2021 highlights the heightened need for economic adjustment mechanisms when fiscal policy adjustments are restricted. The observed exacerbation of capital decline and subsequent output reduction necessitate an increase in public deficit to maintain budget equilibrium, with the increase in public debt being more pronounced compared to 2007. To stimulate the economy, the social planner reduces interest rates, which inadvertently spurs additional borrowing and exacerbates debt levels. To counteract this surge in public debt, the social planner strategically opts to reduce inflation, aligning its trajectory with the evolving economic landscape.

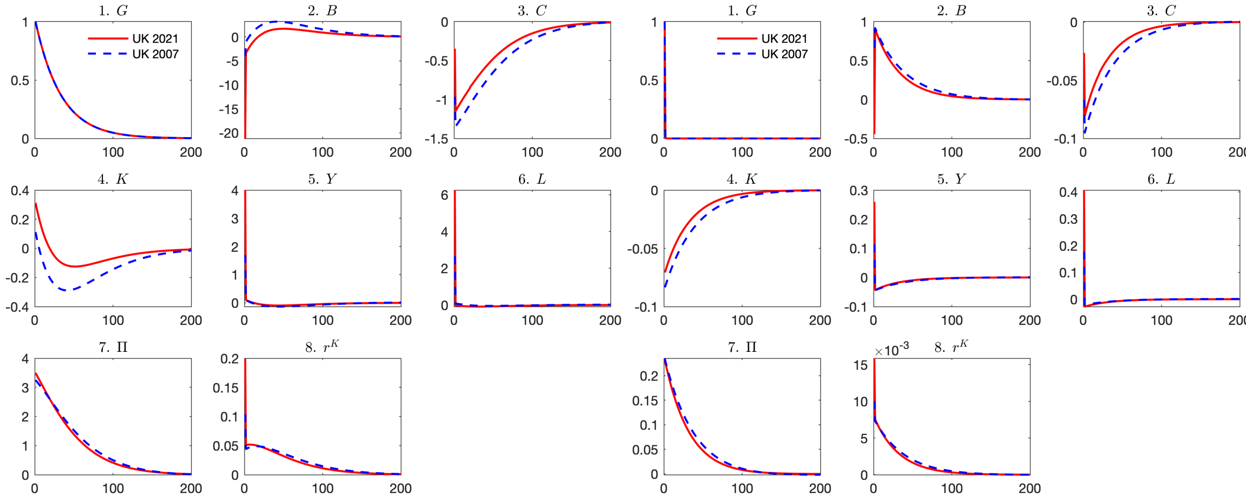
Similar dynamics are observed when analyzing negative productivity shocks, as depicted in Figures 5 and 4. Despite the presence of a negative productivity shock in the latter, the response dynamics echo those observed previously, with a pronounced increase in public debt and subsequent adoption of debt deflation measures to realign debt costs with the steady state. In both scenarios, the optimal path for inflation deviates from the previously assumed zero inflation strategy, emphasizing the evolving nature of optimal inflation policy as an adjustment mechanism in response to economic shocks.

4.5.2 Model Dynamics with Monetary rules

The results obtained so far shed light on the importance of prices as an adjustment mechanism from an optimal perspective, particularly in scenarios where fiscal tools cannot be optimally chosen but are allowed to adjust within the business cycle. This is especially pertinent in the current economic landscape characterized by high debt levels, where optimal fiscal tools indicate the need for higher tax adjustments to accommodate shocks. Given the possibility that the planner may be constrained in adjusting taxes, as discussed in Section 4.5, or alternatively, considering the scenario from the perspective of a central bank lacking control over fiscal tools but possessing discretion over inflation, we proceed to solve the problem outlined by (22)–(36) by closing the first-order conditions (FOCs) related to the choice of fiscal policy parameters, as given by equations (47), (48), and (49). Here, fiscal policy instruments are maintained at their steady-state values: $(\tau_t^k, \tau_t, \kappa_t) = (\tau_t^k, \tau, \kappa)$ as specified in Table 2. Additionally, we introduce a monetary policy rule in the following format:

$$\tilde{R}_t^n = R^n + \phi_{\Pi,t}(\Pi_t - 1), \tag{63}$$

where $\phi_{\Pi,t}$ is the coefficient of the Taylor-rule.



(a) High persistence (b) Low persistence

Figure 6: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a positive spending shock with varying levels of persistence and defined monetary rule.

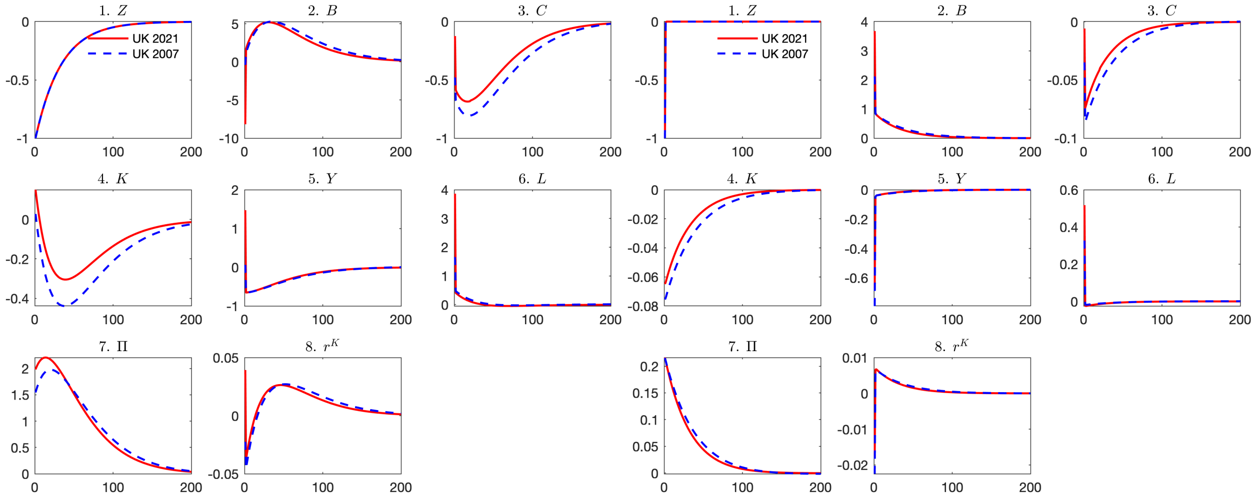
Figure 6 plots the dynamics of public debt B, consumption C, capital K, output Y, labor L, inflation Π, and interest rate r^k in proportional deviations, for two levels of persistence. Panel (a) reports results for the high-persistence case ($\rho_z = 0.97$) and panel (b) reports results for the low-persistence case ($\rho_z = 0.01$).

The observed effects are akin to those depicted in Figure 2, where capital declines in both scenarios for the UK economy following a positive government spending shock, prompting the social planner to consider increasing public debt to facilitate consumption smoothing. Notably, when shock persistence is low, the planner opts for increased public debt, as discussed previously. However, without the ability to adjust taxes to compensate for rising public debt, the planner resorts to inflation as an adjustment mechanism. This reflects a willingness to tolerate inflation as a consequence of increased output resulting from the positive government spending shock, ultimately accepting a period of inflation without taking immediate measures to counter it.

When shock persistence is higher, inflation at the outset to accommodate the shock is approximately 4%, significantly higher than the 0.2% observed when shock persistence is low. Notably, in this scenario, public debt does not increase initially, as inflation effectively reduces the real value of government debt, allowing the government to repay its debt at a lower value. As a result, public debt initially decreases, reflecting an improvement in the government's fiscal position. However, as the inflation mechanism continues and the outstanding value of debt decreases, public debt increases. The higher initial inflation is a consequence of the larger government shock magnitude.

In summary, increasing inflation following a positive government spending shock can be viewed as a policy tool to facilitate economic recovery, alleviate debt burdens, stimulate aggregate demand, and mitigate the risks associated with deflationary pressures.

Next, we shift our focus to analyzing the outcomes in response to a negative productivity shock.



(a) High persistence (b) Low persistence

Figure 7: We compare the performance of the UK economy in two distinct periods: 2021 (red solid line) and 2007 (blue dashed line) following a negative productivity shock with varying levels of persistence and defined monetary rule.

The results closely resemble those obtained in Figure 6. In the scenario with a high-persistence shock, the initial negative productivity shock prompts the social planner to counteract the possible decline in capital by decreasing the interest rate, resulting in an upsurge in capital and inflation. This negative shock generates a wealth effect, leading agents to increase labor supply, contributing to an initial increase in output followed by a subsequent reduction.

Since there is an increase in capital initially, there is no immediate need for public debt to rise. However, the elevated inflation reduces the servicing cost of outstanding debt, eventually leading to an increase in public debt to offset the decrease in capital caused by the higher interest rate.

All trajectories depicted represent optimal paths from the perspective of the social planner, and the heightened inflation in the initial periods following the negative technology shock is deemed optimal. It serves as a mechanism facilitating economic adjustment and shock accommodation in the initial periods. The persistence of the shock directly influences the inflation rate, with higher persistence necessitating a higher inflation rate to accommodate the shock. This is because higher inflation is required to lower the real value of outstanding debt during periods of low capital. Conversely, in the case of low shock persistence, capital declines immediately, and even with the social planner's decision to decrease the interest rate, it is insufficient to boost capital. Consequently, to achieve smooth consumption and increase savings, the social planner opts to increase public debt. Inflation then increases to alleviate the burden of public debt. Unlike the persistent shock scenario, here, even the increase in labor supply is not enough to initially increase output as a form of mitigating the shock.

Comparing Figures 6 and 7, differences in the inflation path for different shock types are evident: demand-driven (resulting from an increase in G) and supply-driven (affecting the Total Productivity Factor Z). Although responses are similar in both cases, the optimal inflation path following a supply shock indicates a need for an initial increase in inflation. This adjustment is optimal because, in the case of a supply shock, it is optimal to reduce interest rates to stimulate the economy, which consequently impacts inflation.

5 Concluding remarks

The economic landscape post-COVID-19 and the energy crisis has accentuated the pivotal role of government intervention in mitigating shocks and fostering economic recovery. Analyzing optimal fiscal and monetary policy responses in the United Kingdom, particularly in scenarios marked by divergent debt-to-GDP ratios observed in 2007 and 2021, reveals crucial insights into managing economic uncertainties.

Firstly, we observe that under a high debt-to-GDP ratio scenario, the dynamics of public debt response to shocks undergo significant changes compared to scenarios with lower debt ratios. This underscores the importance of considering debt levels when designing policy responses to economic shocks.

Secondly, our analysis highlights the role of inflation as an effective adjustment mechanism in the absence of fiscal tools. In scenarios where fiscal instruments are limited, accepting moderate inflation may be optimal to mitigate the adverse effects of shocks and stabilize the economy. Indeed, our findings suggest that policymakers should take a nuanced approach to inflation dynamics when crafting policy responses, especially in situations where fiscal tools are limited or ineffective. While controlling inflation is generally a key objective of monetary policy, our analysis highlights scenarios where accepting moderate inflation may be optimal.

By recognizing the potential benefits of accepting moderate inflation, policymakers can effectively manage economic shocks and promote stability. However, it's crucial to emphasize that this approach must be carefully calibrated and contingent upon thorough analysis of the prevailing economic conditions.

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Appendix