Inventories matter for the transmission of monetary policy: uncovering the cost-of-carry channel*

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Abstract

By setting interest rates, monetary policy affects the cost of carrying inventories. We build a model to capture the resulting "cost-of-carry channel" of monetary policy, finding it to yield interesting non-linearities. We begin with a static model showing that higher inventory costs drive firms, especially those with larger inventories, to reduce prices. Extending this to a dynamic model where firms face distinct demand shocks, we first show in a tractable model with only high and low demand shocks that firms with inventories reduce prices as carrying costs rise, while others may increase them. In aggregate, higher inventory levels make prices more responsive to tighter monetary policy. Finally, a quantitative model with firm heterogeneity and idiosyncratic demand shocks reveals that prices should be more sensitive to the stance of monetary policy when inventory levels are higher (effectively leaving sellers with less market power). By drawing on data from the US housing market, we are able to test this hypothesis – finding strong support for the cost-of-carry channel. Central banks may therefore wish to pay close attention to inventory levels, as they could matter for the strength of monetary policy transmission to inflation.

JEL-classification: D52, E30, E31, E32, E52, E58.

Key words: inventories, monetary policy, monetary transmission mechanism, inflation.

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1 Introduction

In this paper, we develop an inventory model to capture the cost-of-carry channel of monetary policy. This captures the notion that, by setting interest rates, the central bank also shapes the costs of carrying inventories – and thereby firms' incentives to do so. Since changing prices is one way to go about inventory management, there may well be important links from the cost of carrying inventories, to inflation dynamics. In addition, change in inventories affect the demand side effect of the economy, and thus price dynamics through indirect effects.

From a narrative point of view, both commentators and businesses themselves frequently refer to inventory levels when discussing pricing decisions. As for example noted by Robinson (2022), in an *Investors Chronicle* article aptly titled "Interest rates could spell trouble for inventories, liquidity, and IPOs":

> *Carrying costs can rise appreciably as interest rates climb, a worrisome prospect given that they can represent an estimated 25-30 per cent of overall inventory value.*
> *This has become a more acute issue since the pandemic, as companies accelerated the trend towards building resilience into supply chains by increasing inventory levels. Nike is merely the latest big retailer to warn that, along with unfavourable currency movements, its earnings have come under pressure through the inventory-build it undertook following the pandemic and the* ***subsequent discounts aimed at alleviating the situation****. Part of the sportswear giant's profit shortfall will be linked to increased carrying costs.* [Emphasis added]

While the above quote mentions Nike, other companies have also alluded to their inventory levels in relation to their pricing decisions. Trudell (2024) for example mentions how "Tesla is slashing prices (...) in a bid to clear its biggest-ever stockpile. (...) Tesla is offering the deals after producing 46,561 more vehicles than it delivered in the first quarter, adding more cars to inventory than ever before". Similar considerations have been raised in relation to retailers,[1] consumer goods firm Unilever (Dominguez 2023), while Williamson (2023) notes how "destocking policies are meanwhile adding to the downturn in pricing power."

It is interesting to note the timing of these articles, in particular how they post-date the recent rise in interest rates (with some of them explicitly referring to this development). Consulting firms specializing in inventory management frequently point out that "lean" inventory management becomes more consequential when rates are higher. SAFIO Solutions, for example, notes on their website:[2] "As interest rates increase (...) the costs of carrying excess inventory will be increasing as well, impacting your company's bottom line." It then goes on to mention that "capital costs are typically the largest portion of total carrying costs. Capital costs represent the cash that is

[1] See CBS (2022), which is titled "Target's profit craters after it cut prices to clear inventory", and Reuters (2022): "U.S. retailers' ballooning inventories set stage for deep discounts".

[2] See https://safiosolutions.com/increasing-interest-rates-carrying-excess-inventory-can-have-an-even-greater-effect-on-your-cash-flow/. Very similar points are made by Rackbeat (a provider of warehouse management systems), in a post titled "How an Inventory System Helps You Counteract the Red Hot Interest Rate" (https://rackbeat.com/en/how-an-inventory-system-helps-you-counteract-the-increased-interest-rates/).

being tied up in the inventory. These costs include the money spent on the inventory, interest paid on the purchase, and the opportunity cost of the money invested in the inventory rather than other investments like mutual funds." It is furthermore striking how many Western companies only adopted "just-in-time" inventory management when interest rates started soaring in the early 1980s ("post-Volcker"), even though the idea had been around for decades (Petersen 2002).

Next to goods markets, carrying costs may also be relevant in commodity markets – an insight going back to (at least) Deaton & Laroque (1992, 1995, 1996). Frankel (2008*a*,*b*, 2014) , in particular, makes the case that the rate of interest is an important driver of oil prices (with higher interest rates providing greater incentives to economize on oil inventories, which raises available supply, thus depressing the price). In line with this fact, Miranda-Pinto et al. (2023) document how commodity prices tend to fall in response to a US monetary policy tightening – to a degree that is increasing in the storability of the commodity. This supports the cost-of-carry channel, over a parallel general equilibrium channel that operates by slowing down aggregate demand via more conventional transmission channels (Miranda-Pinto et al. 2024).

In the empirical part of this paper (Section 6), we document that very similar forces seem at play for housing inventories in the US housing market. There, we will show that monetary policy has a stronger effect on the cost of housing when the housing inventory (the fraction of homes that is unoccupied) stands at a higher level. Episodes during which many homes are vacant can be thought of as environments in which landlords (or homeowners looking to sell their property) have less market power. When combined with a higher interest rate (which is also the opportunity cost of keeping the property unoccupied on one's balance sheet), this gives landlords a greater incentive to cut their price. They do so in order to speed up the process of getting the property occupied and capitalize on the higher interest rate.

The fact that the cost-of-carry channel has been shown to matter to commodity prices as well as to the cost of housing (in addition to anecdotal evidence, offered earlier in this introduction, suggesting that it matters for storable goods), makes it of direct relevance to a major share of the consumption basket in most countries.[3] This makes the channel arguably worthy of more attention than it has received up to this point. Indeed, it is not mentioned in standard treatments of the monetary transmission mechanism (see, e.g., Boivin et al. (2010)).

In this paper, we analyze the cost-of-carry channel, focusing on how rising interest rates incentivize firms to reduce inventory holdings. While prior studies have touched on this, our contribution lies in examining how increased carrying costs affect prices, and we further innovate by incorporating firm heterogeneity into the debate, arguing that this heterogeneity is a key factor that endogenizes firms' inventory decisions. Specifically, firms face distinct demand shocks, which generate idiosyncratic variation in how inventories are accumulated over time.[4]

[3] In the US, the shelter component accounts for over 30% of the CPI basket. For PCE, the housing-related share stands at over 15% – lower, but still substantial. While the *direct* share of commodity prices is lower, they are an important driver of price dynamics via their prevalence throughout the supply chain.

[4] The heterogeneous firm environment offers two main advantages. First, it allows us to study an economy with a cross-sectional distribution of inventories, which can be used to better calibrate the model. Second, while the total variance of the demand shock is currently exogenous, we plan to endogenize it in future work, so that variance increases during recessions and decreases during booms. This would enable us to study the effects of uncertainty shocks at the firm level more effectively.

The essence of our argument is captured by a simple, static model, where we show that as inventory carrying costs rise, firms are incentivized to lower their prices to economize on inventory holdings. Firms with higher inventory levels when a shock occurs are more exposed to this dynamic and thus have a stronger incentive to cut prices. To examine equilibrium considerations and quantitatively assess the significance of the various forces involved, we then develop a tractable model in which firms encounter two levels of demand shocks: high and low. Firms in a low-demand period know they will face high demand in the next period, resulting in a finite distribution of firms at equilibrium. This setup allows us to study how firms' pricing decisions depend on their inventory levels. At the start of each period, firms receive an idiosyncratic demand shock and produce goods one period in advance.[5] They carry over inventories by paying a carrying cost, and their pricing decisions determine sales and the inventories held for the next period. Lastly, firms choose production at the end of each period.

In equilibrium, high-demand firms indeed carry no inventory, while low-demand firms do. The intuition is straightforward: firms in a low-demand period know they will face high demand in the next period, so they carry inventories to meet that demand. This simplified equilibrium provides analytical expressions for firms' price decisions, especially those carrying inventories, and shows how these decisions are influenced by monetary policy, which affects the cost of carrying inventories or the firm's pricing kernel.

The main lesson of the tractable model is that firms with inventories lower their prices when carrying costs rise, boosting sales and reducing inventories for the next period, while firms without inventories raise their prices. Aggregating these decisions shows that higher inventory levels make aggregate prices more responsive to tighter monetary policy, highlighting the role of the cost-of-carry channel. In other words, when inventory levels are high, an increase in carrying costs leads to a larger price reduction.[6]

To validate this result, we test it within a quantitatively relevant environment where firms face varying uninsurable idiosyncratic demand shocks, leading to a distribution of firms with different inventory levels. We develop a model that accounts for this full distribution with firms differing in their resources and demand risks, and facing costly price adjustments.[7] Solving this model poses challenges, as the distribution of resources represents a state variable of infinite dimension. Using a state space representation, projection, and perturbation methods inspired by Reiter (2009), we confirm that the findings from the tractable model hold.[8]

The model includes: monopolistically competitive wholesale firms, monopolistically

[5]These are idiosyncratic shocks, and the theory remains agnostic about their nature. We refer to them as demand shocks for simplicity.

[6]In Section 3.4, we demonstrate that many of our main results hold in a model similar to Bils & Kahn (2000), where inventories facilitate sales by allowing firms to draw from stock in response to unexpected demand. In this complete markets environment, inventories are part of the technology, and there is no market failure. An advantage of our framework is that inventories emerge as a firm-level distortion. While this paper is not normative, we characterize the positive effects, leaving room for potential normative applications.

[7]In other words, we relax the assumption of having only two states of the world—high and low demand shocks—which limited the number of firm types.

[8]While solving this in sequence space was an option, the state-space representation proves more efficient in this context and enhances the comparison with the simpler model.

competitive sectoral firms, a perfectly competitive retail firm, a representative household, and the central bank. Wholesale firms use labor provided by households for production. The framework is discrete and infinite in time, with firms producing goods one period ahead and incorporating inventories to meet sectoral demand. The central bank is responsible for managing monetary policy.

To maintain consistency with the earlier tractable model, we assume that individual firms (wholesale firms) set their prices based on their idiosyncratic shocks, while sectoral firms face nominal frictions modeled as costly price adjustments. This framework, known as HANK, follows the seminal work of Kaplan et al. (2018), and more specifically the treatment of heterogeneous firms in New Keynesian environments, as in Ottonello & Winberry (2020). By introducing these additional layers of heterogeneity and new channels, we enhance the analysis of monetary policy in this context.

The model examines how firms respond to two key shocks: an increase in inventory carrying costs and a negative TFP shock. When the cost of carrying inventories rises, firms reduce their inventory holdings to avoid higher expenses. This reduction in inventories leads firms to cut prices to clear existing stock, shifting focus to real-time production, which raises labor demand and wages. A negative TFP shock initially leads firms to hold more inventory as a buffer against lower production, raising prices in the process. However, as production remains low and storage costs rise, firms eventually deplete inventories and lower prices to stimulate sales.

In addition, the transmission channels of both shocks are influenced by the economy's endogenous inventory levels, which result from idiosyncratic firm demand shocks. By introducing a shock to the variance of these firm-specific shocks, we can compare scenarios with high and low inventory levels, as heightened firm risk increases inventories. Applying a contractionary monetary policy shock at two points—when inventory levels are high and when they are low—the model shows that higher inventories lead to a more pronounced price reduction following the monetary policy shock.

The main findings from both the tractable and quantitative models are supported by our empirical analysis of the U.S. housing market, where we show that higher housing inventories (the fraction of unoccupied homes) lead to a greater reduction in prices following contractionary monetary policy.

2 Related literature

The heterogeneous-firms models to analyze the inventory-channel of demand and supply shock is related to three different literatures.

First, inventory dynamics have indeed a rich history in business cycle models, as inventories are typically thought to account for a significant share of fluctuations in GDP (Blinder & Maccini 1991, Fitzgerald 1997, Ramey & West 1999). The idea central to this paper, that higher interest rates give firms a greater incentive to economize on their inventory holdings, has been developed before (see, e.g., Lieberman (1980), Irvine (1981), Blinder (1981), Akhtar (1983), Maccini et al.

(2004)). While some of the aforementioned papers offered empirical support for this hypothesis,[9] other papers (such as Ramey (1989)) have failed to do so, which has contributed to the theory's declining popularity over time.

Armed with recent progress in monetary policy shock identification, we revisit this debate. When doing so, we deviate from the earlier literature along two dimensions. First, informed by our model (presented in Sections 3 and 4) as well as aided by improved data availability, we broaden our focus to also look at the response of prices. This contrasts with the aforementioned earlier literature, which solely looked at outcomes in firms' inventory holdings (which is challenging to measure – especially at a high frequency, whereas analyzing outcomes at a lower frequency might bias results). Our model, however, suggests that the price response should vary with inventory levels, which is a hypothesis that is arguably easier to test than looking at observed changes in inventory holdings.[10] Second, we focus on one particular market, namely that for housing. That one, we argue in Section 6, is more suited for testing our theory's core hypothesis. When proceeding along these lines, we find strong support for the cost-of-carry channel.[11]

This paper also relates to the study of heterogeneous firms within New Keynesian frameworks. Some papers, such as Andrés & Burriel (2018), examine optimal monetary policy in this context, designing policies that account for heterogeneity in total factor productivity and strategic price interactions among firms. Adam & Weber (2019) demonstrate that incorporating heterogeneous firms and systematic trends in firm-level productivity alters predictions for the optimal inflation rate. Additionally, González et al. (2020) argue that central banks should implement monetary expansion following a TFP shock to alleviate constraints on firms. Other studies focus on the significance of firm dynamics within heterogeneous firm models to understand aggregate fluctuations and the effects of macroeconomic policies, as seen in Hopenhayn (1992), Hopenhayn & Rogerson (1993), Erosa & González (2019), Bartelsman et al. (2013), Clementi & Palazzo (2016), and Sedlácek & Sterk (2019). Furthermore, this paper contributes to the literature that emphasizes the importance of accounting for firm heterogeneity in business cycle models, as developed by Melitz (2003), Ghironi & Melits (2005), and Bilbiie et al. (2012). We contribute to this literature by showing that inventories are a critical driver of inflation dynamics and must be considered in discussions of optimal monetary policy. Specifically, our findings suggest that after a negative TFP shock, inflation dynamics will

[9] Additional support is presented in papers focusing on the credit channel of monetary policy. Gertler & Gilchrist (1994) find that especially smaller firms' inventory holdings are sensitive to borrowing costs, while Kashyap et al. (1993) and Kashyap et al. (1994) make the same point for firms that are bank-dependent.

[10] With respect to the level of inventory holdings, the theory predicts that higher interest rates should lower inventories (as maintaining them becomes costlier). However, when all firms attempt to shed their inventories by cutting prices (leaving their relative prices unchanged), inventory holdings might show relatively little movement in aggregate. Especially when the intertemporal substitution elasticity on the consumer side is low and/or when they become less willing to hold inventories too as interest rates rise. In this example, one would still see the cost-of-carry channel operate on prices though, which motivates our focus. Junayed & Khan (2009) also argue that analysis of inventory dynamics is problematic when trying to test the cost-of-carry channel, further strengthening the case of our focus on price responses.

[11] At the same time, we realize that our findings are, strictly speaking, "local" to the housing market. We therefore remain open to the possibility that the cost-of-carry channel has little-to-no significance for the goods market (which has been the focus of most earlier studies). However, given the importance of housing services in price indices (recall footnote 3), we see our results for the housing market as interesting in their own right. On top of that, Frankel (2008*a*,*b*, 2014) and Miranda-Pinto et al. (2023, 2024) have documented empirical support for the cost-of-carry channel in commodity markets – with resulting commodity prices being an important driver of CPI inflation as well.

depend on inventory levels, and thus the optimal monetary policy approach should take this into account.

Finally, and more broadly, the wider inventory literature has mostly evolved around three stylized facts:

1. Production is more volatile than sales;
2. Inventories are procyclical;
3. The ratio of inventories-to-sales is countercyclical (implying that sales display stronger procyclicality than inventories).

While an early literature treated inventories as a way to smooth production over the cycle, this approach has fallen out of favor as it is inconsistent with stylized fact #1 (Blinder 1986, Eichenbaum 1989). Instead, scholars have tried to reconcile (some of) the above stylized facts by modeling inventories as a factor that avoids stock-outs, thereby boosting sales (Kahn 1987, Bils & Kahn 2000), whereas others have approached the issue by focusing on non-convex production costs (leading to "production bunching"; Ramey (1991)) or by taking an (S,s)-type approach (Khan & Thomas 2007). Den Haan & Sun (2024) augment a standard New Keynesian model with a "sell friction", which enables their model to replicate key stylized facts whilst also highlighting the importance of inventories for business cycle fluctuations. Of note, their model also offers a fully microfounded environment in which the cost-of-carry channel arises.

Relative to this last literature, our objective is more focused. We wish to understand the role of inventories in the transmission of aggregate shocks. To do so, we first present our simple theoretical model in Section 3 to gain intuition and ensure tractability of the overall effects. In Section 4, we extend the model to a quantitative framework, suitable for analyzing general equilibrium effects across the entire economy. Section 5 discusses the main findings of the quantitative model and examines the overall equilibrium effects. Section 6 offers an empirical test of its key prediction on the US housing market. Finally, Section 7 concludes the paper.

3 The Simple Model

The core of our argument can be captured by a very simple model, which is essentially static in nature. Consider a profit-maximizing firm that controls its production level (y), the price it charges (p), and the level of inventories it chooses to maintain (x). Both production and inventories incur quadratic costs, specifically a production cost and a carrying cost, respectively, and goods produced will only be available for sale in the future (not explicitly modeled here but addressed in further detail below). The firm's problem can be represented as:

$$
\begin{aligned}
&\max_{p,y} pS(p) - \psi_y \frac{y}{2}^2 - \psi_x \frac{x}{2}^2 + \vartheta q, && (1)\\
&s.t.\ x = x_0 - S(p),\\
&\quad q = x + y,\\
&\quad x \geq 0,
\end{aligned}
$$

where $S(p)$ is the demand function, assumed to be continuous, three-times differentiable, and with $S'(p) < 0$. The cost of producing y units is given by $\psi_y \frac{y^2}{2}$, and the cost of carrying x units in inventory by $\psi_x \frac{x^2}{2}$. The final term in the objective function, $+\vartheta q$, serves as a shorthand to represent the positive value of carrying goods over into the future, where $\vartheta > 0$, and q, the sum of end-of-period inventories and production, represents the number of goods available for sale in the future. The firm begins with x_0 units in inventory, carried over from the unmodelled past, meaning that it will have $x = x_0 - S(p)$ units left in inventory after selling $S(p)$ units. When setting its price p, the firm considers the relationship between the price it charges, the quantity of goods it will sell, and, consequently, the amount of inventory it will need to carry forward (subject to a quadratic cost governed by ψ_x).

Solving the problem described by (1) leads a profit-maximizing firm to set its optimal production and price as follows:

$$
y = \frac{\vartheta}{\psi_y}, \tag{2}
$$

$$
0 = [p + \psi_x(x_0 - S(p)) - \vartheta]\, S'(p) + S(p). \tag{3}
$$

The production component of the model (2) is intentionally simplified, allowing us to focus on price setting as governed by the implicit function in (3). In particular, we are interested in understanding how a firm's "exposure" to inventory carrying costs—reflected in its initial inventory level, x_0 (carried over from the past)—influences its pricing strategy when faced with changes in inventory carrying costs, ψ_x.

Proposition 1. (Price setting) *As the cost of carrying inventories rises, profit-maximizing behavior induces the firm to lower its price, i.e.:*

$$
\frac{\partial p}{\partial \psi_x} < 0,
$$

more so the greater its pre-existing inventory level x_0.

The proof is in Appendix A. Proposition 1 conveys the logic, frequently alluded to by many firms (recall, for example, the quotes featured in the Introduction), that as the costs of carrying inventories rise, firms gain an incentive to lower their prices in an attempt to economize on their inventory holdings. Firms carrying more inventory when the shock hits (i.e., firms with higher x_0) are more exposed to this channel and thus have the strongest incentive to cut their prices.

The model environment presented so far is highly simplified, static, and partial in nature—serving purely to illustrate the main mechanism at play. In what follows, we will develop a full quantitative model—centering on the dynamic version of the problem described by (1)—to explore the impact of equilibrium considerations and quantitatively assess the importance of the various forces involved. In particular, we aim to explain why firms carry inventories and how their pricing decisions vary based on their inventory levels.

Given that interest rates are a crucial determinant of inventory carrying costs, it is essential to incorporate an explicit role for monetary policy in this context. Section 4 will address this, drawing further implications for the monetary transmission mechanism, which will help us understand how these factors interact within the economy.

To achieve this, we now consider an economy where a final good, S_t, is produced by a unique profit-maximizing representative firm. This firm combines intermediate goods, $S_{i,t}$, produced by different firms indexed by $i \in [0,1]$, using a standard Dixit-Stiglitz aggregator with an elasticity of substitution denoted by γ:

$$S_t = \left(\int_0^1 \varepsilon_{i,t}^{\frac{1}{\gamma}} S_{i,t}^{\frac{\gamma-1}{\gamma}} \, d_i \right)^{\frac{\gamma}{\gamma-1}}, \tag{4}$$

where $\varepsilon_{i,t}$ is an idiosyncratic shock affecting the sales of individual firm i, which can be interpreted as a demand shock for firm i's variety.[12] For any intermediate good $i \in [0,1]$, production $S_{i,t}$ is carried out by a monopolistic firm and sold at price $p_{i,t}$.

Profit maximization by the firm producing the final good implies:

$$S_{i,t} = \frac{1}{p_{i,t}^{\gamma}} \varepsilon_{i,t} P_t^{\gamma} S_t. \tag{5}$$

Finally, the price index is defined as:

$$P_t = \left(\int_0^1 p_{i,t}^{1-\gamma} \varepsilon_{i,t} \, d_i \right)^{\frac{1}{1-\gamma}}. \tag{6}$$

Now, we focus on the problem of individual firms in a partial equilibrium analysis. First, observe that we can rewrite equation (5) for a specific firm i, omitting the subscript i, in the following form:

$$S_t(p_t) = \bar{S}(p_t)\varepsilon_t. \tag{7}$$

Here, ε_t represents the demand shock in period t, and $\bar{S}(p_t)$ denotes the firm's demand as a function of its price p_t, with $S'(p_t) < 0$ indicating that demand decreases as the price increases. Finally, total sales are given by $S_t(p_t)$.

The timeline of the model, outlining the decisions made by individual firms, is as follows:

1. Production takes one period.

[12] These idiosyncratic shocks can be associated with demand shocks, though the theory is agnostic about their nature, which could also be interpreted as productivity shocks.

2. At the start of period t, the firm is impacted by a demand shock, which affects the quantity of goods sold, $S_t(p_t)$.

3. To meet the demand $S_t(p_t)$, the firm relies on the previous period's production, y_{t-1}, and the total inventory available at the beginning of period t, denoted by x_t. Thus, the total quantity of goods available for sale at the start of period t is:

$$q_t = y_{t-1} + x_t. \tag{8}$$

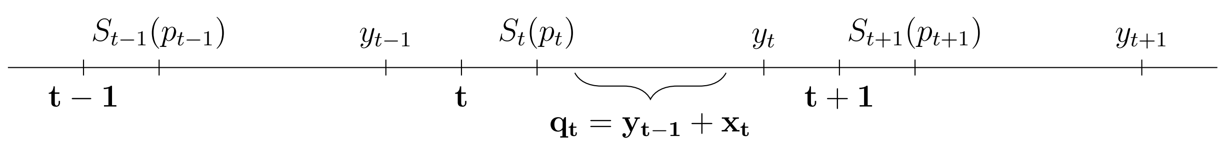
4. After the demand shock is realized, the firm sets the selling price for its goods and chooses the amount of inventory to hold. The inventory available in the next period is:

$$x_{t+1} = q_t - S_t(p_t). \tag{9}$$

5. A holding cost of inventories, denoted by ψ_x, is incurred.

6. At the end of period t, the firm decides on the quantity to be produced, denoted as y_t.

The timeline of the model can be summarized as follows:



Therefore, the firm's problem in nominal terms at period t can be stated as:

$$V(q_t) = \max_{\{p_t, y_t\}} p_t S_t(p_t) - \psi_y \frac{y_t^2}{2} - \psi_x \frac{x_{t+1}^2}{2} + \beta \mathbb{E}\left[V(q_{t+1})\right], \tag{10}$$

subject to:

$$q_{t+1} = q_t - S_t(p_t) + y_t, \tag{11}$$

$$x_{t+1} = q_t - S_t(p_t) \geq 0, \tag{12}$$

where $\psi_y \frac{y_t^2}{2}$ is the production cost of y_t. We impose that holding inventories incurs a cost, where the total cost is given by $\psi_x \frac{x_{t+1}^2}{2}$. The operator $\mathbb{E}$ is taken with respect to the demand shock. Equation (11) represents the law of motion for the quantity available in period $t+1$. Finally equation (12) is the inventory constraint, assumed to be positive.

In recursive formulation the problem above can be written as:

$$V(q) = \max_{\{p, y\}} pS(p) - \psi_y \frac{y^2}{2} - \psi_x \frac{(q - S(p))^2}{2} + \beta \mathbb{E}\left[V(q')\right], \tag{13}$$

subject to:

$$q' = q - S(p) + y, \tag{14}$$

$$x' = q - S(p) \geq 0. \tag{15}$$

For the sake of simplicity, we make the following assumption regarding monetary policy in this Section.[13]

Assumption A. *Monetary policy affects either ψ_x or β, where $\beta = \frac{1}{R^N}$ is the firm's pricing kernel, and R^N is the nominal interest rate.*

This assumption implies that if we let ψ_x be an increasing function of the interest rate, then higher values of ψ_x will reduce the incentive to hold inventories. Consequently, the firm may choose to lower prices to increase the quantity of goods sold in the current period, t, thereby reducing the amount of inventory carried into the next period. A similar rationale applies when considering a reduction in β due to an increase in the interest rate. In what follows, we introduce a simple structure for the demand shocks affecting this economy, which will further clarify these results

Let μ denote the Lagrange multiplier associated with the inventory constraint defined by equation (15). The solution to the firm's problem is obtained by solving the system represented by equations (16) through (19).[14]

$$\frac{S(p) + pS'(p)}{S'(p)} + \psi_x(q - S(p)) = \psi_y y + \mu, \tag{16}$$

$$\psi_y y = \beta \mathbb{E}\left[-\psi_x(q' - S(p')) + \psi_y y' + \mu'\right], \tag{17}$$

$$q' = q - S(p) + y, \tag{18}$$

$$\mu(q - S(p)) = 0. \tag{19}$$

3.1 Reduced Heterogeneity Equilibrium

We aim to establish an equilibrium framework that generates a distribution of inventories based on the history of idiosyncratic shocks. For now, we would like to simplify this structure to better understand the mechanisms underlying individual firms' decisions, enabling us to aggregate these firms to derive the industry equilibrium.

To achieve this goal, consider an economy with a straightforward shock structure. We will assume there are two types of shocks: a High-Demand shock (H) and a Low-Demand shock (L), denoted as:

$$\varepsilon = \begin{cases} 1 + \Delta & \text{if shock } = H, \\ 1 - \Delta & \text{if shock } = L. \end{cases}$$

This structure allows us to analyze how these demand shocks influence individual firm decisions and the overall industry equilibrium.

In the scenario of a High-Demand shock (H), there's a probability of α for the economy to remain in the H state, with the remaining probability $(1 - \alpha)$ indicating a transition to the Low-Demand state (L). Conversely, when the economy is in the L state, the probability of transitioning

[13] We override this assumption in Section 4 to incorporate an explicit role for monetary policy in this context.

[14] In Appendix B.1, we derive the results stated by these equations.

to the H state is 1, while the probability of staying in the L state is 0.[15] We summarize these transition probabilities in the following transition matrix:

$$\Pi = \begin{bmatrix} \pi(H|H) & \pi(L|H) \\ \pi(H|L) & \pi(L|L) \end{bmatrix} = \begin{bmatrix} \alpha & 1-\alpha \\ 1 & 0 \end{bmatrix}. \tag{20}$$

Considering this reduced heterogeneity equilibrium structure, we employ a guess-and-verify approach to find an equilibrium where High-Demand firms do not hold inventories, while Low-Demand firms do. In this equilibrium, three types of firms will emerge, as discussed below.

Using the notation $x_{IJ} = x(I|J)$, where x_{IJ} denotes the realization of variable x in state I given the state J, Proposition 2 summarizes the equilibrium conditions in this model. In this equilibrium, the prices will behave as stated in Proposition 2.

Proposition 2. (Equilibrium conditions) *In the case of a High-Demand shock, the firm sets prices such that it does not hold inventories, i.e., $S(p_H) \geq q_H$. Thus, the firm ensures:*

$$S(p_{HH}) = q_{HH} \quad and \quad S(p_{HL}) = q_{HL}.$$

Regardless of the previous state (High or Low demand), there exists a price at which the available quantity can satisfy the demand, ensuring market equilibrium under these conditions. Specifically, the equilibrium conditions are given by:

$$p_{HH} = S^{-1}(q_{HH}), \tag{21}$$

$$p_{HL} = S^{-1}(q_{HL}), \tag{22}$$

$$\frac{S(p_{LH})}{S'(p_{LH})}(1 + E^S_{p_{LH}}) + \psi_x(q_{LH} - S(p_{LH})) = \psi_y y_{LH}, \tag{23}$$

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_{LH} = \beta\frac{S(p_{LH})}{S'(p_{LH})}(1 + E^S_{p_{LH}}) + \beta\frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) + \beta\psi_x q_{HL}, \tag{24}$$

$$(\psi_y + \beta\psi_x)y_{HL} = \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH})) + \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}, \tag{25}$$

$$(\psi_y + \beta\psi_x)y_{HH} = \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH})) + \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}, \tag{26}$$

$$\mu_{HH} = \frac{S(p_{HH})}{S'(p_{HH})}(1 + E^S_{p_{HH}}) - \psi_y y_{HH}, \tag{27}$$

$$\mu_{HL} = \frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) - \psi_y y_{HL}. \tag{28}$$

Proposition 3. (Existence of Equilibrium) *Given the conditions outlined in Proposition 2, an*

[15] This simple structure will allow us to obtain equilibrium conditions that will depend only on the histories (HH, HL, LH).

equilibrium exists as long as the following conditions are satisfied:

$$\frac{S(p_{HH})}{S'(p_{HH})}(1+E^S_{p_{HH}}) > \psi_y y_{HH},$$
$$\frac{S(p_{HL})}{S'(p_{HL})}(1+E^S_{p_{HL}}) > \psi_y y_{HL},$$
$$q_{LH} > 0.$$

The first two conditions imply that firms in the High-Demand state do not hold inventories, while the third condition, $q_{LH} > 0$, ensures that firms in the Low-Demand state carry inventories.

The proof is provided in Appendix B.2. It is important to note that the equilibrium conditions hold as long as the demand function $\bar{S}$ admits an inverse. When the current state is Low-Demand (L), equation (23) implies that $\mu_{LH} = 0$, meaning firms will carry positive levels of inventories in this state.

From equations (27) and (28), we observe that firms do not hold inventories during High-Demand shocks as long as the corresponding Lagrange multipliers are positive, as stated in Proposition 3. Therefore, in this simplified structure, firms stockpile inventories in anticipation of future High-Demand shocks when they are in a Low-Demand state. This inventory accumulation allows them to meet the expected future demand.

In this model, the rationale for firms holding inventories is primarily driven by demand shocks.

Result 1. *Assume the demand function is given by:*

$$\bar{S}(p) = \frac{1}{p^\gamma},$$

where γ is the demand elasticity and $\eta = \left(\frac{\gamma-1}{\gamma}\right)$ is the market power. In the steady state, there are three types of firms producing total quantities q_{HH}, q_{LH}, and q_{HL}, and their decisions to produce quantities y_H and y_L depend only on the current level of demand. In such an equilibrium, the values of y_H and y_L are determined by:

$$\psi_y y_H = \beta(1-\alpha)y_L(\psi_x+\psi_y) - \beta\psi_x(1-\alpha)\left(\frac{\beta\eta}{\psi_y y_L}\right)^\gamma(1+\Delta) + \beta\alpha\eta\left(\frac{1+\Delta}{y_H}\right)^{\frac{1}{\gamma}}, \tag{29}$$

$$(\psi_x+\psi_y)y_L = \left(\frac{(1-\Delta)(\psi_y y_L)^\gamma}{(\psi_y y_L)^\gamma(y_L+y_H) - (\beta\eta)^\gamma(1+\Delta)}\right)^{\frac{1}{\gamma}}\eta + \psi_x\left(\frac{\beta\eta}{\psi_y y_L}\right)^\gamma(1+\Delta). \tag{30}$$

The corresponding prices for the different firm types are given by:

$$p_{HH} = \left(\frac{1+\Delta}{y_H}\right)^{\frac{1}{\gamma}}, \tag{31}$$

$$p_{HL} = \left(\frac{1+\Delta}{y_L + y_H - p_{LH}^{-\gamma}(1-\Delta)}\right)^{\frac{1}{\gamma}}, \tag{32}$$

$$p_{LH} = \left(\frac{(1-\Delta)(\psi_y y_L)^{\gamma}}{(\psi_y y_L)^{\gamma}(y_L + y_H) - (\beta\eta)^{\gamma}(1+\Delta)}\right)^{\frac{1}{\gamma}}. \tag{33}$$

Finally, the total quantities produced by the different firm types are given by:

$$q_{HH} = y_H, \tag{34}$$

$$q_{LH} = y_H, \tag{35}$$

$$q_{HL} = y_H + y_L - \frac{1}{p_{LH}^{\gamma}}(1-\Delta). \tag{36}$$

The proof is provided in Appendix B.3.

Note that when $\gamma = 1$, implying $\eta = 0$, we encounter conditions such as $y_L = y_H = 0$. However, in this scenario, the prices become indeterminate. Consequently, such a situation cannot represent an equilibrium and we need to impose $\gamma > 1$.

Result 2. *Let y_H and y_L be functions of ψ_x, and let p_{LH} be defined by*

$$p_{LH} = \left(\frac{(1-\Delta)(\psi_y y_L)^{\gamma}}{(\psi_y y_L)^{\gamma}(y_L + y_H) - (\beta\eta)^{\gamma}(1+\Delta)}\right)^{\frac{1}{\gamma}}.$$

Then, around the steady-state values $y_{H,0}$ and $y_{L,0}$ of y_H and y_L respectively, p_{LH} is decreasing with respect to ψ_x if the sum of the derivatives of y_L and y_H with respect to ψ_x is positive.

The proof is detailed in Appendix B.4. This model demonstrates a distinctive characteristic: due to its structure, when firms are in a low-demand state and experience a demand shock, they anticipate transitioning to a high-demand state in the future. Consequently, if the cost of holding inventories increases, firms will reduce their current inventory levels. To compensate and prepare for future demand, firms in the low-demand state will increase their production. If this increase in production exceeds the reduction in the high-demand state, the result stated above follows. This behavior becomes more apparent when analyzing the model's dynamics.

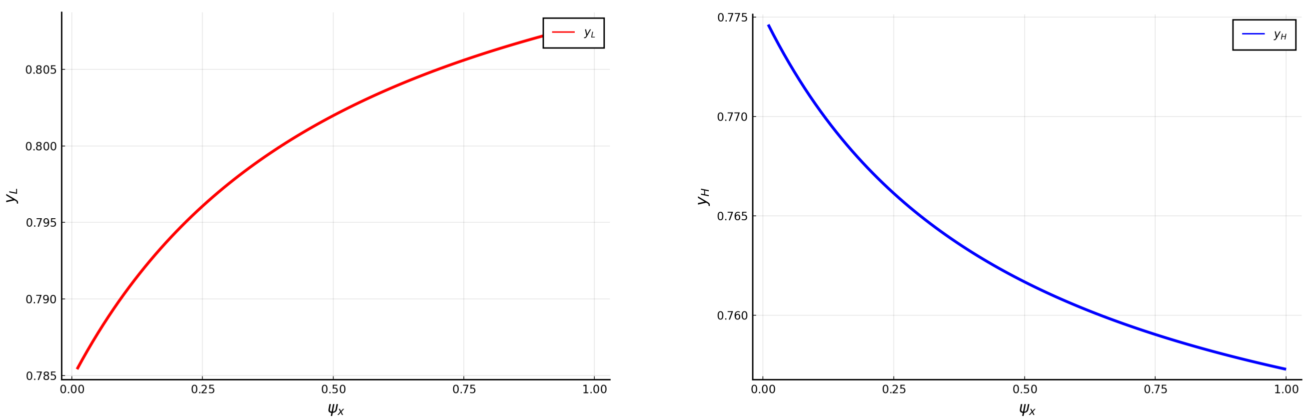
3.2 Equilibrium conditions

To further illustrate the results discussed above, we present a numerical example demonstrating the existence of this equilibrium. For this purpose, we refer to the parameters outlined in Table 1:

Figures 1, 2, 3, and 4 show the equilibrium conditions for different values ψ_x.

Parameter	Description	Value
ψ_x	Cost to carry inventories	0.5
ψ_y	Cost to produce	0.7
α	Probability to continue in High-Demand	0.3
β	Discount factor	0.95
Δ	Demand shock	0.2
γ	Elasticity of demand with respect to price	2.0
η	Market power	0.5

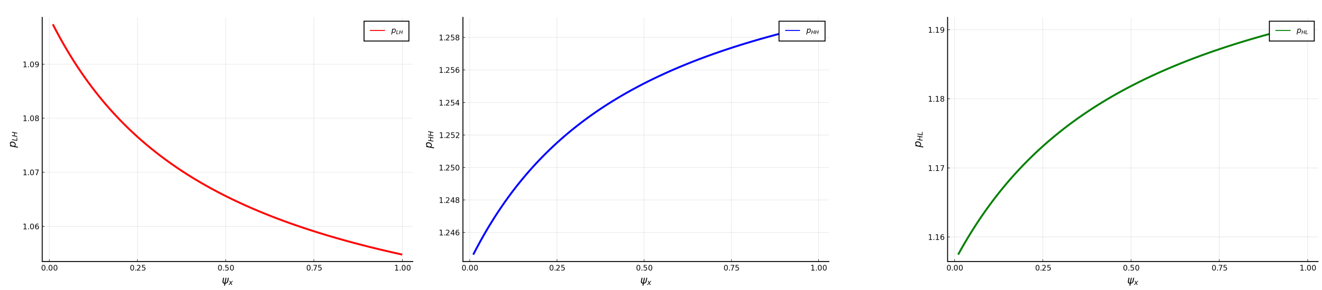
Table 1: Model Parameters.



(a) Production in the Low-Demand state (L).

(b) Production in the High-Demand state (H).

Figure 1: Production for different values of ψ_x.



(a) The price in L, considering the previous state was H.

(b) The price in H, considering the previous state was H.

(c) The price in H, considering the previous state was L.

Figure 2: Prices for different values of ψ_x.

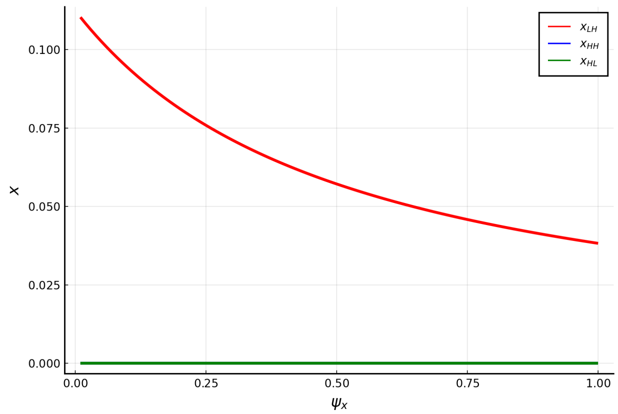
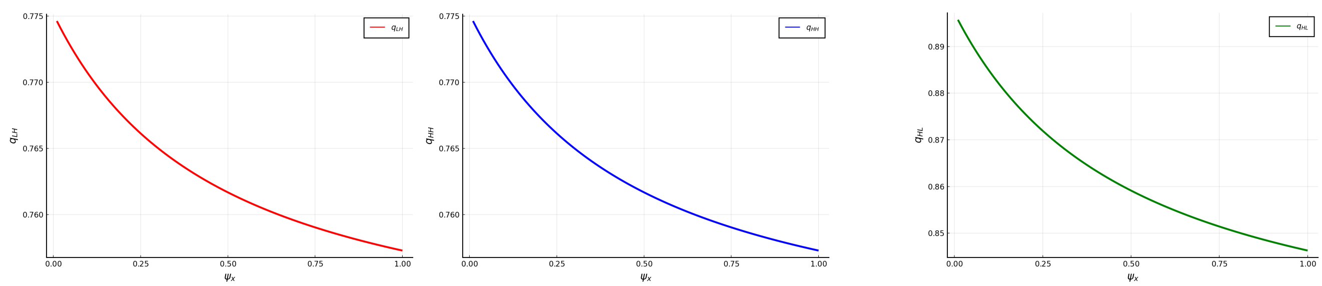


Figure 3: Inventories across various states and different values of ψ_x.



(a) The quantity in L, considering the previous state was H.

(b) The quantity in H, considering the previous state was H.

(c) The quantity in H, considering the previous state was L.

Figure 4: Quantities for different values of ψ_x.

Observe that during periods of low demand, the firm must carry over some of the available goods as inventory into the next period. Given the associated costs, higher inventory expenses lead to lower prices for the firm's products, as illustrated in Panel (a) of Figure 2. This pricing strategy is adopted because, in such scenarios, a lower price results in an increase in sales volume, thereby reducing the quantity of goods that need to be carried over.

In contrast, firms in the high-demand state raise their prices. This increase is a response to higher holding costs and reduced production. The scarcity of the product, due to decreased production, justifies the higher price, which is passed on to consumers, as shown in Panels (b) and (c) of Figure 2.

It is important to note that adopting $\beta = \frac{1}{R^N}$ demonstrates the same logic. Specifically, it shows that an increase in the interest rate leads to a reduction in the price charged by the firm in a low-demand state, as the cost-effectiveness of carrying inventories into the future diminishes. Therefore, whether considering inventory carrying costs or the variable β, we observe similar pricing strategic behavior from the firm.

Notice that as ψ_x increases, there is a significant shift in production strategies. Specifically, production in the low-demand state (y_L) increases, while production in the high-demand state (y_H)

decreases. This shift reflects firms' adjustments in response to anticipated future demand.

An increase in ψ_x leads to higher inventory holding costs. In response, firms increase production in the low-demand state as a precautionary measure. This strategy involves producing more to build up inventory in anticipation of future high demand. By doing so, firms aim to mitigate the increased costs associated with holding stock, as illustrated in Panel (a) of Figure 1. [16] Moreover, the reduction in prices leads to an increase in sales, which corroborates the need for increased production.

Conversely, production in the high-demand state decreases. This reduction occurs because firms raise their prices, leading to decreased sales, which justifies a lower production level, as seen in Panel (b) of Figure 1.

The firm only carries inventories when demand is low, according to the equilibrium conditions we highlighted in Proposition 2. The quantity of inventories carried is given by:

$$x' = \begin{cases} 0 & \text{if state } = HH, \\ 0 & \text{if state } = HL, \\ \left(\frac{\beta\eta}{\psi_y y_L}\right)^{\gamma}(1+\Delta) - y_L & \text{if state } = LH. \end{cases}$$

It is evident that higher costs associated with maintaining inventories result in a lower quantity of inventories, as shown in Figure 3. Similarly, this principle applies to the variable β: as the interest rate increases, the value derived from holding inventories decreases.

In Appendix C, we present equilibrium conditions for different values of ψ_y. The results indicate that increased production costs lead to a reduction in production in both states, an increase in prices to offset the higher costs, and a decrease in inventories and the total quantity held, as expected. Appendix D presents equilibrium conditions for lower values of γ. As anticipated, the higher γ, the greater the absolute value of the price elasticity of demand, meaning that demand becomes more elastic; thus, small changes in price result in significant changes in quantity demanded. Conversely, a lower γ signifies that consumers are less responsive to price changes, which grants firms more pricing power. Consequently, with a lower γ, prices tend to be higher for the same set of parameters. In such cases, firms exercise more power, leading to lower production levels and, as a result, lower quantities held. Thus, as illustrated in Figures in Appendix D, a lower γ allows the monopolist to set higher prices due to increased market power.

As a final note, observe that this model setup can generate state-dependent, real effects of monetary policy, even when prices are fully flexible. It demonstrates that incentives to carry inventories are influenced by the ex-ante real interest rate. Specifically, when expected inflation is high (which corresponds to a lower ex-ante real rate), there is a stronger incentive to accumulate inventories. High expected inflation effectively provides a return for producing now and holding onto the goods.

One way to accumulate inventories is to raise prices. This strategy can bring expected inflation into the present, even without price stickiness. In other words, the presence of inventories

[16] This will become clearer once we analyze the dynamics of this economy.

can make price setters forward-looking, as they anticipate future conditions and adjust their pricing strategies accordingly.

3.3 Dynamics of the partial equilibrium

Parameter	Description	Value
ψ_x	Cost to carry inventories	0.5
ψ_y	Cost to produce	0.7
α	Probability to continue in High-Demand	0.8
β	Discount factor	0.95
Δ	Demand shock	0.5
γ	Elasticity of demand with respect to price	2.0
η	Market power	0.5

Table 2: Model Parameters.

We now present the dynamic responses of each type of firm to a monetary policy shock affecting ψ_x, using the calibration parameters provided in Table 2. To facilitate comparison and highlight the differences between business cycles across scenarios, we maintain the same shock magnitude in both cases. Figure 5 shows the Impulse Response Functions (IRFs) for a persistent increase in ψ_x, demonstrating its impact on production, prices, inventories, and quantities across different states.[17]

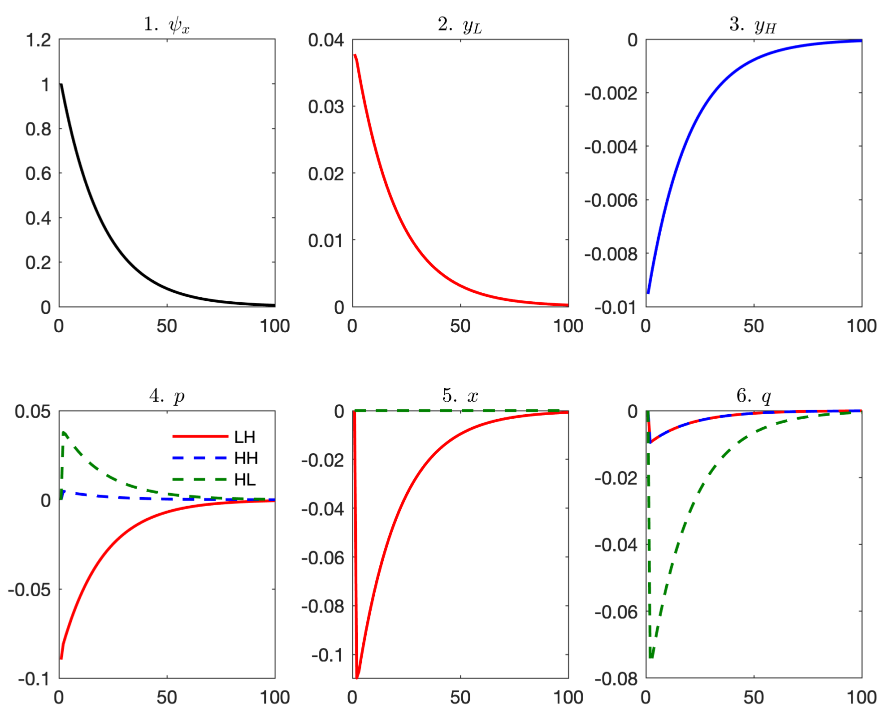


Figure 5: Comparison of Impulse Response Functions for selected variables following a positive shock in ψ_x.

[17]In Appendix E.1, we provide detailed results for each case individually.

Notice that each panel reports the proportional change for the variable under consideration, in percentage points. For instance, Panel 1 reports a persistent increase in ψ_x for 100 periods, after an increase of 1 % on impact, except for the inventories, for which we compute the absolute variation.

We observe that when ψ_x increases, the firm's production behavior varies depending on whether it is in a low- or high-demand state. In the low-demand state (y_L), production rises as firms anticipate future high demand (Panel 2 of Figure 5). This precautionary increase is driven by the expectation of transitioning to a high-demand state. Firms produce more during low demand to mitigate future inventory costs. Additionally, as shown in Panel 4 of Figure 5, when inventory costs increase, firms lower prices to reduce excess inventory, boosting sales and creating incentives to produce more.

In contrast, in the high-demand state (y_H), production decreases (Panel 3 of Figure 5). Firms rely on the inventory accumulated during the low-demand phase, allowing them to reduce production and avoid the costs of meeting demand with fresh output.

Pricing strategies also adjust following a positive shock to ψ_x. In the low-demand state, firms lower prices to cut inventory holding costs, quickly selling off excess stock. Conversely, in the high-demand state, firms raise prices due to higher holding costs and reduced production, which increases scarcity and justifies the price hike.

In essence, as ψ_x increases, the firm's cost structure changes, prompting adjustments in production and inventory strategies. This mirrors earlier observations: during low demand, firms must carry over inventory, and higher costs lead to lower prices to increase sales. In high demand, firms raise prices to compensate for higher holding costs and reduced production.

This logic also applies to changes in $\beta = \frac{1}{R^N}$, as shown in Appendix E.2. When the interest rate rises, the cost-effectiveness of holding inventories decreases, leading to similar outcomes: lower prices during periods of low demand and higher prices during high demand, with more pronounced effects. However, one notable difference is that production in the low-demand state will also decline.

Second-order moments of this economy are presented in Appendix E.3, which confirm that economic volatility increases under a preference shock. However, apart from production, the effects remain largely similar.

3.4 Discussion

It is crucial to highlight that introducing demand shocks helps endogenize firms' decisions to carry inventories, providing a rationale for their inventory choices. The primary focus here is to illustrate that in an environment with inventories, an increase in inventory carrying costs leads firms to reduce prices to manage their stockpile.

Our discussion aims to justify why firms carry inventories from an endogenous perspective. For comparison, consider a model similar to Bils & Kahn (2000), where inventories facilitate sales by allowing firms to draw down stock in response to unexpected demand, avoiding additional production costs.

In this alternative model, we define the demand function as:

$$S_t(p_t, x_t) = \bar{S}(p_t)x_t^{\xi}, \tag{37}$$

where, unlike the previous specification, holding inventories now boosts total demand. The model's timeline and structure remain consistent. Here, a firm's sales increase with higher relative inventory holdings and decrease with higher relative prices. This environment can also be seen as treating inventories as a taste shifter, where firms use them to capture additional demand, similar to Kryvtsov & Midrigan (2010).

In Appendix F, we present the model and demonstrate that our main results hold, particularly that an increase in inventory carrying costs leads to reduced prices. Our baseline model offers the advantage of allowing firms to make inventory decisions based on idiosyncratic shocks, whereas this specification involves a representative firm using inventories to boost demand. Here inventories facilitate sales by allowing firms to draw from stock in response to unexpected demand.

The results are consistent, with the added benefit of directly observing that increased inventory carrying costs lead to lower prices in a representative firm setting. In this complete markets environment, inventories are part of the technology, and there is no market failure. An advantage of our framework is that inventories emerge as a firm-level distortion. While this paper is not normative, we characterize the positive effects, leaving room for potential normative applications.

Appendix G also explores a combined model where demand is given by:

$$S_t(p_t, x_t) = \bar{S}(p_t)x_t^{\xi}\varepsilon_t, \tag{38}$$

with shocks structured as follows:

$$\varepsilon = \begin{cases} 1 + \Delta & \text{if shock } = H, \\ 1 - \Delta & \text{if shock } = L. \end{cases}$$

In this combined environment, similar to the previous models, firms in a low-demand state will produce more than those in a high-demand state. However, production levels will show a U-shaped pattern across different values of ψ_x. This reflects how the relationship between inventory holding costs (ψ_x) and production levels (y_L and y_H) depends on firms' inventory management and production planning in response to carrying costs.

When ψ_x is low, holding inventory is inexpensive, so firms maintain larger inventories, allowing steady production with fewer adjustments. Higher prices and lower demand lead to reduced production and reliance on inventory.

When ψ_x is high, holding costs are significant, prompting firms to minimize inventory by reducing prices, which increases demand and necessitates higher production to meet that demand directly. This scenario highlights the stronger influence of inventory levels on pricing and production responses.[18]

[18] See Figure 39 in Appendix G.

3.5 Industry dynamics

Given the same shock structure as previously discussed in equation (20) in Section 3.1, we know the steady-state numbers of each type of firm are:

$$n_{HH} = \alpha \left(\frac{1}{2-\alpha}\right), \quad n_{HL} = (1-\alpha)\left(\frac{1}{2-\alpha}\right), \quad n_{LH} = \left(\frac{1-\alpha}{2-\alpha}\right). \tag{39}$$

Using this information, along with Equation (4), which represents the final good in this model with reduced heterogeneity, we obtain in equilibrium:

$$S = \left(n_{HH}\varepsilon_{HH}^{\frac{1}{\gamma}} S_{HH}^{\frac{\gamma-1}{\gamma}} + n_{HL}\varepsilon_{HL}^{\frac{1}{\gamma}} S_{HL}^{\frac{\gamma-1}{\gamma}} + n_{LH}\varepsilon_{LH}^{\frac{1}{\gamma}} S_{LH}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}. \tag{40}$$

Assume there is a representative consumer who provides labor to firms. Under the same framework as the Simple Model, we set the wage $w = \frac{\psi_y}{2}$. The consumer provides labor services to firms and consumes the aggregate profits of individual firms. Total consumption in the steady state is:

$$C = S - \frac{\psi_x}{2}\left(n_{HH}x_{HH}^2 + n_{HL}x_{HL}^2 + n_{LH}x_{LH}^2\right). \tag{41}$$

Using the results from Proposition 2 and setting the price of the final good as the numeraire ($P = 1$), the equilibrium conditions follow directly. According to our specification of shocks, a higher Δ leads to higher total inventories in the economy. Next, we examine the system's dynamics following a monetary policy shock that affects ψ_x, using the same parameters as in Table 2, except for Δ, where we compare two cases: one with a higher $\Delta = 0.5$, representing a higher aggregate level of inventories, and one with a lower $\Delta = 0.1$, representing a lower level of aggregate inventories.

Figure 6 illustrates the impulse response functions for a persistent increase in ψ_x on prices, consumption, inventories, quantity, and GDP. Each panel in Figure 6 shows the proportional change in percentage points for the corresponding variable. For example, Panel 1 reports a persistent increase in ψ_x over 100 periods, after an increase of 1% on impact.

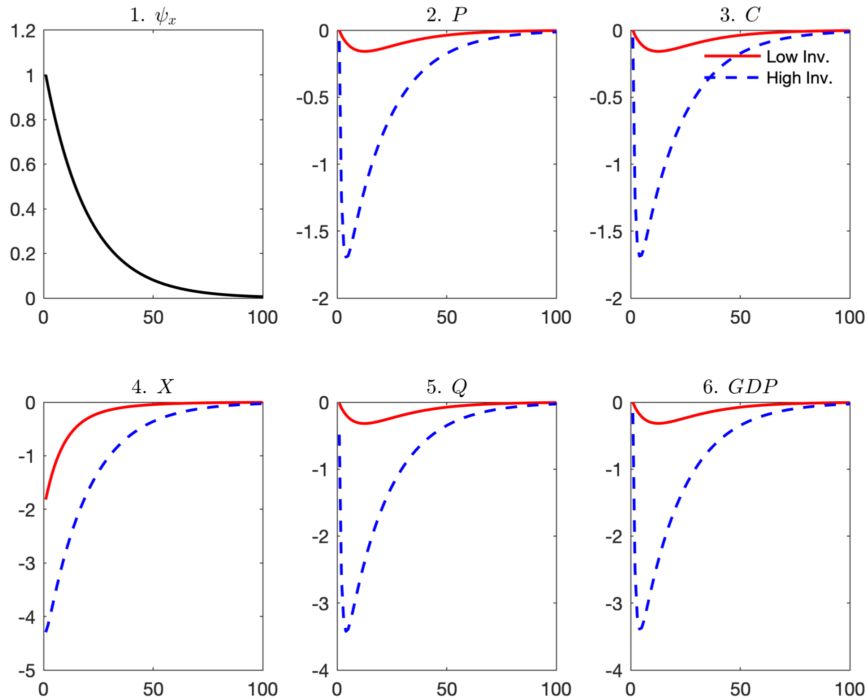


Figure 6: Comparison of Impulse Response Functions for selected variables following a positive shock in ψ_x, with two different inventory levels: Higher and Lower.

Notice in Figure 6 that the higher the level of inventories, for the same type of shock, the greater the reduction in prices. For the lower inventories case, we consider a scenario where the total inventories level is close to zero, which demonstrates that in this situation, the effect on aggregate prices is minimal. This highlights the crucial role of inventories and their carrying cost as a transmission mechanism to prices after shocks.

Table 3 summarizes the first and second moments of the key variables for each case analyzed. For each variable, we report the steady-state value, labeled as "Mean," and the normalized standard deviation in percentage terms, which is the standard deviation divided by the mean, referred to as "Std."

		Low Inv.	High Inv.
P	Mean	1.000	1.000
	Std	0.007	0.061
C	Mean	0.700	0.753
	Std	0.007	0.060
X	Mean	0.005	0.064
	Std	0.042	0.140
Q	Mean	0.705	0.820
	Std	0.015	0.122
GDP	Mean	0.700	0.756
	Std	0.015	0.121
		Correlations	
corr(X,P)		0.6277	0.9484
corr(X,X_{-1})		0.9057	0.9524
corr(GDP,GDP_{-1})		0.9961	0.9782

Table 3: First and second moments for key variables under different inventory levels: Low and High.

4 Quantitative assesment

We now solve for the evolution of inventories and prices over the business cycle in a quantitatively relevant environment. In the first part of the paper, we made some assumptions regarding demand shocks, allowing us to study price behavior after monetary policy shocks that affect the cost of carrying inventories in a simple environment. In this section, we relax some assumptions, particularly the assumption that there are only two states of the world (H and L), which previously resulted in an economy with a limited number of firm types. We also introduce endogenous labor in the production function and a representative household. By incorporating a more complex structure for idiosyncratic demand shocks for firms, we generate a time-varying joint distribution of inventories and price decisions.

Previously, we observed that contractions in monetary policy, indicated by an increase in inventory carrying costs, led to a reduction in the total quantity of inventories, primarily achieved through price reductions. The objective of this quantitative assessment is to determine whether this result holds true when analyzed within a general equilibrium model.

4.1 Model

This model is a quantitative generalization of the one presented previously. In this framework, due to demand shocks, firms will have persistent differences in their inventory levels and, consequently, different pricing decisions. These differences will lead to varied reactions to monetary policy changes. All firms draw demand shocks from the same distribution. There are five types of agents in this economy: a continuum of monopolistically competitive firms (wholesale firms), a continuum of monopolistically sectoral firms, a representative perfectly competitive firm (retail firm) that buys the production of the sectoral firms, a representative household, and the central bank. Unlike the previous model, individual wholesale firms use labor for the production of the good y, which is supplied by the households. As before, the production to be used in period t will be produced at the end of period $t-1$, and together with the inventory levels, it composes the quantity of goods q_t, which can then be used to meet the demand of the sectoral firm. Time is discrete and infinite. Each period, there is a fixed mass of heterogeneous firms distributed on an interval $\mathcal{I}$, and a fixed mass of sectoral firms distributed on an interval $\mathcal{J}$. Households provide labor services to the firms, and the central bank implements monetary policy.

4.1.1 Risk structure

Below we discuss the risk strucuture of the model.

Idiosyncratic demand shock. The risk structure is the same as in the previous section. Specifically, we assume that idiosyncratic demand shocks follow an AR(1) process, such that the idiosyncratic level of demand for the product of firm i at time t is given by $\varepsilon_{i,t} = \rho_\varepsilon \varepsilon_{i,t-1} + u^\varepsilon_{i,t}$, where ρ_ε represents the persistence of the idiosyncratic demand shock and $u^\varepsilon_{i,t} \sim_{IID} \mathcal{N}(0, \sigma^2_\varepsilon)$. Each idiosyncratic demand shock $\varepsilon_{i,t}$ can take E distinct values in the set $\mathcal{E} \subset \mathbb{R}^+$.

Technology and TFP shock. In this section, we also assume an aggregate productivity risk, which is modeled as an AR(1) process $z_t = \rho_z z_{t-1} + u^z_t$, with ρ_z being the persistence parameter and $u^z_t \sim_{IID} \mathcal{N}(0, \sigma^2_z)$. The aggregate productivity, denoted Z_t, is related to z_t through the following functional form: $Z_t = Z_0 e^{z_t}$.

4.1.2 Firms

The production side of the economy consists of a continuum of monopolistically competitive firms, indexed by i, distributed over an interval $\mathcal{I}$. These firms sell their goods to another monopolistically competitive firm, indexed by j, which operates under similar market conditions and is distributed over an interval $\mathcal{J}$. The goods sold by the firms in sector i are aggregated by a sectoral firm using an aggregator with an elasticity of substitution between different products, denoted by γ. This sectoral firm then sells the aggregated goods to a retail firm, which further aggregates them into a final good, S_t, using another aggregator with an elasticity of substitution between different sectors, denoted by γ_j.

Wholesale firms. The model follows the timing and structure discussed in the Simple Model in Section 3, where each firm i in the wholesale sector operates under a monopolistically competitive structure. We assume each firm's production decisions for period t are made in period $t-1$. At the start of period t, each firm i is impacted by a demand shock $\varepsilon_{i,t}$, which affects the demand for goods sold by the firm according to the following equation:

$$S_{i,t} = \frac{1}{p_{i,t}^{\gamma}} \varepsilon_{i,t} P_{j,t}^{\gamma} S_{j,t}, \tag{42}$$

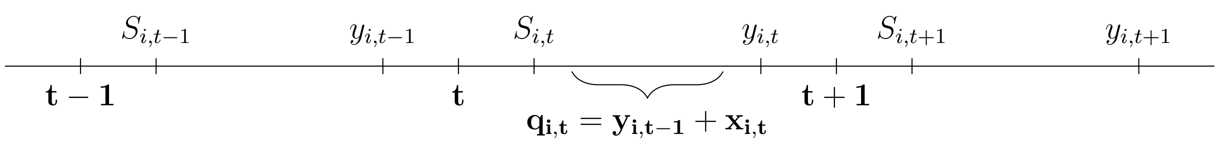
where $P_{j,t}$ is the price charged by the sectoral firm j, and $S_{j,t}$ is the Dixit-Stiglitz aggregator of differentiated products (i.e., the production of the sectoral firm j). To meet this demand, firm i depends on the production level $y_{i,t-1}$ and the total inventory available at time t, denoted by $x_{i,t}$. Given this, the total quantity of goods available to be sold at the beginning of period t is:

$$q_{i,t} = y_{i,t-1} + x_{i,t}. \tag{43}$$

As before, at the end of period t, the firm determines the quantity to be produced, denoted as $y_{i,t}$. After the demand shock is realized and the firm sets the selling price for its goods, we can calculate the inventories available in the next period as:

$$x_{i,t+1} = q_{i,t} - S_{i,t}. \tag{44}$$

The timeline of the model for each firm i can be summarized as follows:



Production. Different from the simple model, we now introduce labor into the production function. Additionally, we allow an aggregate productivity shock to enter the production function. Each firm i produces the same good $y_{i,t}$ using labor $l_{i,t}$ with the following production function, for $t \geq 0$:

$$y_{i,t} = Z_t l_{i,t}^{\alpha}, \tag{45}$$

where Z_t is the economy-wide level of productivity and $\alpha \in (0, 1)$ is the labor share.

Firm Heterogeneity. Given the whole discussion above, it is evident that at the beginning of each period, a firm is characterized by its predetermined quantity q and the current demand shock ε. Therefore, the aggregate state of the economy is defined by the distribution of firms (q, ε) and by the aggregate shock Z.

We can summarize the distribution of firms over (q, ε) using the probability measure Λ, which is defined on the Borel algebra generated by open subsets of the product space $Q \times E$. The evolution of this distribution over time is governed by a mapping Γ, such that:

$$\Lambda' = \Gamma(\Lambda, Z).$$

Here, Λ' denotes the updated distribution of firms, and Γ represents the mapping function that describes how the distribution evolves given the current state Λ and the aggregate shock Z. This distribution depends on the firms' decisions, such as their production and pricing strategies, as well as exogenous factors affecting these decisions, such as the cost of carrying inventories.

Sectoral firms. The output in sector j is produced by a monopolistically competitive firm that combines the sales of individual firms, $S_{i,t}$, where $i \in \mathcal{I}$, using a standard Dixit-Stiglitz aggregator with an elasticity of substitution denoted by γ:

$$S_{j,t} = \left(\int_0^1 \varepsilon_{i,t}^{\frac{1}{\gamma}} S_{i,t}^{\frac{\gamma-1}{\gamma}} d_i \right)^{\frac{\gamma}{\gamma-1}}, \tag{46}$$

where $S_{j,t}$ represents the aggregate sales to firm in the sector j, $S_{i,t}$ denotes the specific sales of firm i, $\varepsilon_{i,t}$ represents the demand shocks for each firm i in period t, and $\gamma > 1$ is the elasticity of substitution.[19]

Solving the cost-minimization problem of the firm in sector j implies that:

$$S_{i,t} = \frac{1}{p_{i,t}^{\gamma}} \varepsilon_{i,t} P_{j,t}^{\gamma} S_{j,t},$$

which is exactly the demand function that the wholesale firm will take as given, as represented by equation (42), where $p_{i,t}$ denotes the price charged by firm i. Associated with this problem we have a price index for the output in sector j denoted by:

$$P_{j,t} = \left(\int_0^1 p_{i,t}^{1-\gamma} \varepsilon_{i,t} d_j \right)^{\frac{1}{1-\gamma}}. \tag{47}$$

Thus, an indivual firm's sales are thus increasing in its demand shock and decreasing in its relative price.

Retail firm. The retail firm aggregates the sales of the sectoral monopolistically competitive firms using the following aggregator:

$$S_t = \left(\int_0^1 S_{j,t}^{\frac{\gamma_j-1}{\gamma_j}} d_j \right)^{\frac{\gamma_j}{\gamma_j-1}}, \tag{48}$$

where S_t represents the aggregate sales, $S_{j,t}$ denotes the specific sales of sector j, and γ_j is the elasticity of substitution.

Solving the cost-minimization problem of the final firm implies that:

$$S_{j,t} = \frac{1}{P_{j,t}^{\gamma_j}} P_t^{\gamma_j} S_t,$$

where $P_{j,t}$ denotes the price charged by firm j. The price P_t is the price index for aggregate sales S_t

[19] We impose $\gamma > 1$ to ensure positive marginal revenue.

and can be expressed as:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\gamma_j} d_j \right)^{\frac{1}{1-\gamma_j}} . \tag{49}$$

Problem of the sectoral firm. In addition to the production costs associated with acquiring goods from individual firms, represented by $\int_0^1 p_{i,t} S_{i,t} d_i$, we assume that sectoral firms j face a quadratic price adjustment cost à la Rotemberg (1982) when setting their prices. Following the litterature, this price adjustment cost is proportional to the magnitude of the sector j's relative price change and is given by $\frac{\kappa}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 S_t$, where $\kappa \geq 0$. Letting P_t denote the the price index of the economy, we can thus express the real profit of sector j at date t, denoted by $\Omega_{j,t}$, as:

$$\Omega_{j,t} = \left(\frac{P_{j,t}}{P_t}\right) S_{j,t} - \frac{\int_0^1 p_{i,t} S_{i,t} d_i}{P_t}(1-\tau^S) - \frac{\kappa}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 S_t - t_t^S, \tag{50}$$

where t_t^S is a lump-sum tax used to finance the subsidy τ^S.

Sectoral firm j sets the price schedule $(P_{j,t})_{t\geq 0}$ to maximize the intertemporal profit as of date 0:

$$\max_{(P_{j,t})_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{M_t}{M_0} \Omega_{j,t} \right],$$

where β is the discount factor of the households (since households own the sectoral firms) and $\frac{M_t}{M_0}$ is the pricing kernel, with $M_t = U'(C_t)$. Note that $\frac{\int_0^1 p_{i,t} S_{i,t} d_i}{P_t} = \xi_{i,t}\left(\frac{P_{j,t}}{P_t}\right)^{\gamma} S_{j,t}$, where $\xi_{i,t} = \int_0^1 \left(\frac{p_{i,t}}{P_t}\right)^{1-\gamma} \varepsilon_{i,t} d_i$.

The sector's optimization problem is therefore:

$$\max_{(P_{j,t})_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{M_t}{M_0} \left[\left(\left(\frac{P_{j,t}}{P_t}\right)^{1-\gamma_j} - \xi_{i,t}\left(1-\tau_t^S\right) \left(\frac{P_{j,t}}{P_t}\right)^{\gamma} \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma_j} \right) S_t - \frac{\kappa}{2}\left(\frac{P_{j,t}}{P_{j,t-1}} - 1\right)^2 S_t - t_t^S \right].$$

The first-order condition for this problem yields:

$$\begin{aligned} &\left((1-\gamma_j)\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma_j} + (\gamma_j - \gamma)\xi_{i,t}\left(1-\tau_t^S\right)\left(\frac{P_{j,t}}{P_t}\right)^{-(\gamma_j-\gamma)} \left(\frac{P_{j,t}}{P_t}\right)^{-1} \right) \frac{S_t}{P_t} \\ &- \kappa(\Pi_t - 1)\left(\frac{1}{P_{j,t-1}} P_{j,t} \frac{S_t}{P_{j,t}}\right) + \beta \mathbb{E}_t \left[\frac{M_{t+1}}{M_t} \kappa(\Pi_{t+1} - 1) \frac{P_{j,t+1}}{P_{j,t}^2} \frac{S_{t+1}}{S_t} S_t \right] = 0, \end{aligned}$$

where the gross inflation rate is defined as $\Pi_t = \frac{P_{j,t}}{P_{j,t-1}}$. Simplifying this equation gives:

$$\left((1-\gamma_j) + (\gamma_j - \gamma)\xi_{i,t}\left(1-\tau_t^S\right)\left(\frac{P_t}{P_{j,t}}\right)^{1-\gamma}\right)\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma_j}\frac{1}{P_t}S_t - \kappa(\Pi_t - 1)\Pi_t\frac{S_t}{P_{j,t}}$$
$$+ \beta\kappa\mathbb{E}_t\left[\frac{M_{t+1}}{M_t}(\Pi_{t+1}-1)\Pi_{t+1}\frac{S_{t+1}}{S_t}\right]\frac{S_t}{P_{j,t}} = 0.$$

By seeting $\tau_t^S = \frac{1-\gamma}{\gamma_j - \gamma}$ to achieve an efficient steady state and recognizing that this solution is independent of the sector type j, we can define a symmetric equilibrium where $P_{j,t} = P_t$ for all sectors j. This leads to:

$$\left((1-\gamma_j) + (\gamma_j - \gamma)\xi_{i,t}\left(\frac{\gamma_j - 1}{\gamma_j - \gamma}\right)\right) - \kappa(\Pi_t - 1)\Pi_t + \beta\kappa\mathbb{E}_t\left[(\Pi_{t+1}-1)\Pi_{t+1}\frac{S_{t+1}}{S_t}\frac{M_{t+1}}{M_t}\right] = 0.$$

This result leads to the Phillips curve in this model, given by:

$$\Pi_t(\Pi_t - 1) = \frac{\gamma_j - 1}{\kappa}(\xi_{i,t} - 1) + \beta\mathbb{E}_t\left[\Pi_{t+1}(\Pi_{t+1}-1)\frac{S_{t+1}}{S_t}\frac{M_{t+1}}{M_t}\right]. \tag{51}$$

Finally, observe that the real profit of the sector, which is independent of its type, is given by:

$$\Omega_t = \left(1 - \xi_{i,t} - \frac{\kappa}{2}\left(\Pi_t - 1\right)^2\right)S_t. \tag{52}$$

4.1.3 Household preferences and program

Preferences. Households are expected-utility maximizers with a time-separable utility function and a constant discount factor denoted by $\beta \in (0,1)$. Their period intertemporal utility $U(C_t, L_t)$, defined over aggregate consumption C_t and labor hours L_t, is assumed to be time-separable. Preference shocks are not considered in this quantitative section.

$$U(C_t, L_t) = u(C_t) - v(L_t). \tag{53}$$

The function $u : \mathbb{R}^+ \to \mathbb{R}$ is twice continuosly differentiable, strictly increasing, and strictly concave, with $u'(0) = \infty$, while $v : \mathbb{R}^+ \to \mathbb{R}$ is twice continuosly differentiable, strictly increasing, and strictly convex, with $v'(0) = 0$. Households rank streams of consumption $(C_t)_{t\geq 0}$ and labor $(L_t)_{t\geq 0}$ using the intertemporal utility criterion $\mathbb{E}_0 \sum_{t=0}^{\infty} U(C_t, L_t)$. Firms will maximize their profits, which are returned to their shareholders—the representative household.

Household's program. The resources of the representative household consist of labor income, where the household receives the real wage w_t in period t for each unit of endogenous labor effort L_t supplied, as well as aggregate real profits Θ_t of the individual firms and Ω_t of the sectoral firms.

The household will use these resources to consume according to the following program:

$$\max_{\{C_t,L_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \chi \frac{L_t^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}} \right), \tag{54}$$

$$C_t \leq w_t L_t + \Theta_t + \Omega_t, \tag{55}$$

where $\mathbb{E}_0$ denotes the expectation operator, $\beta \in (0,1)$ is the constant discount factor, $\phi > 0$ is the Frisch elasticity of labor supply, and χ is a parameter that scales labor disutility. The representative household owns all the firms in the economy. The first-order condition for the representative household's maximization problem is:

$$v'(L_t) = w_t u'(C_t). \tag{56}$$

4.1.4 Monetary authority

Assume that households are allowed to trade public debt, with the supply denoted by B_t at date t. The public debt is issued by the government and is assumed to be free of default risk. This nominal debt pays a predetermined nominal gross interest rate. In other words, the interest rate between periods $t-1$ and t is known at $t-1$. Denote this gross nominal interest rate by R_{t-1}^N, with the corresponding real interest rate for public debt given by $\frac{R_{t-1}^N}{\Pi_t}$, where Π_t is the gross inflation rate. Due to inflation, the real rate is no longer predetermined. Under these conditions, there is an additional first-order condition for the household's problem:

$$U'(C_t) = \beta \mathbb{E}_t \left[\frac{R_t^N}{\Pi_{t+1}} U'(C_{t+1}) \right].$$

The nominal interest rate R_t^N is set according to the following simple Taylor rule:

$$\frac{R_t^N}{R^N} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\Pi} \iota_t, \tag{57}$$

where R^N is the steady-state nominal interest rate, Π is the steady-state inflation rate, and $\log(\iota_t) = \rho_\iota \log(\iota_{t-1}) + u_t^\iota$ represents a persistent monetary policy shock. Here, ρ_ι denotes the persistence of the monetary shock, ϕ_Π indicates the response of the nominal interest rate to inflation, and $u_t^\iota \sim_{IID} \mathcal{N}(0, \sigma_\iota^2)$.

4.2 Recursive Competitive Equilibrium

We provide the recursive competitive equilibrium for the household and firm problems. In this formulation, given the aggregate state of the economy (Z, Λ), firms take their individual state (q, ε) and set their price and labor decisions. This means the firm's problem can be written recursively as:

$$
\begin{aligned}
V(q,\varepsilon;Z,\Lambda) = \max_{\{p,l\}} & \frac{p(q,\varepsilon;Z,\Lambda)}{P(Z,\Lambda)} S(p(q,\varepsilon;Z,\Lambda),\varepsilon) - w(Z,\Lambda) l(q,\varepsilon;Z,\Lambda) - \psi_x \frac{(q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon))^2}{2} \\
& + \mathbb{E}\left[\lambda' V(q',\varepsilon';Z',\Lambda')\right], \qquad (58) \\
& \text{subject to:} \\
& q' = q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon) + Z l(q,\varepsilon;Z,\Lambda)^{\alpha}, \qquad (59) \\
& q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon) \geq 0, \qquad (60) \\
& \Lambda' = \Gamma(\Lambda, Z), \qquad (61)
\end{aligned}
$$

where λ' represents the firm's pricing kernel. Since firms are owned by the households, we impose the household pricing kernel, i.e., $\lambda' = \beta \frac{u'(C')}{u'(C)}$. One can observe that the structure of the problem represented by (58) - (61) is similar to the one presented in the simple model (13) - (15). Note that $w(Z,\Lambda) = \frac{\psi_y Z^2}{2}$ if we set $\alpha = 1/2$, and we are in the same structure as in the simple model.

Observe that firms with the same level of idiosyncratic demand shock and the same level of quantity q choose the same price $p(q,\varepsilon;Z,\Lambda)$ and labor demand $l(q,\varepsilon;Z,\Lambda)$. The optimal firm decisions are given by:

$$
\frac{S(p(q,\varepsilon;Z,\Lambda),\varepsilon) + p(q,\varepsilon;Z,\Lambda) S'(p(q,\varepsilon;Z,\Lambda),\varepsilon)}{S'(p(q,\varepsilon;Z,\Lambda),\varepsilon)} + \psi_x (q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon)) = \frac{w(Z,\Lambda) l(q,\varepsilon;Z,\Lambda)^{1-\alpha}}{\alpha Z} + \mu, \qquad (62)
$$

$$
\frac{w(Z,\Lambda) l(q,\varepsilon;Z,\Lambda)^{1-\alpha}}{\alpha Z} = \mathbb{E}\left[\lambda' \left(-\psi_x (q' - S(p(q',\varepsilon';Z',\Lambda'),\varepsilon')) + \frac{w(Z',\Lambda') l(q',\varepsilon';Z',\Lambda')^{1-\alpha}}{\alpha Z'} + \mu' \right)\right], \qquad (63)
$$

$$
q' = q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon) + Z l(q,\varepsilon;Z,\Lambda)^{\alpha}, \qquad (64)
$$

$$
\mu (q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon)) = 0. \qquad (65)
$$

4.2.1 Stationary Recursive Competitive Equilibrium

Our market equilibriim definition can be stated as follows:

Definition 1. *A recursive competitive equilibrium for this model is a collection of individual functions $V(q,\varepsilon;Z,\Lambda)$, $l(q,\varepsilon;Z,\Lambda)$, $p(q,\varepsilon;Z,\Lambda)$, $y(q,\varepsilon;Z,\Lambda)$,$q'(q,\varepsilon;Z,\Lambda)$, $x(q,\varepsilon;Z,\Lambda)$, $S(q,\varepsilon;Z,\Lambda)$, $\pi(q,\varepsilon;Z,\Lambda)$, $\lambda(Z,\Lambda)$, $C(Z,\Lambda)$, $L(Z,\Lambda)$, aggregate quantities $S(Z,\Lambda)$, $X(Z,\Lambda)$, $Q(Z,\Lambda)$, $\Pi(Z,\Lambda)$, $\Theta(Z,\Lambda)$, $\Omega(Z,\Lambda)$, aggregate price $P(Z,\Lambda)$, price process $w(Z,\Lambda)$, and a law of motion for the distribution of firms $\Lambda' = \Gamma(\Lambda, Z)$, such that, for an initial quantity q and distribution of demand shocks, the following conditions hold:*

1. *Given $\lambda(Z,\Lambda)$, $w(Z,\Lambda)$, $P(Z,\Lambda)$, $S(Z,\Lambda)$, Z', and Λ', the functions $V(q,\varepsilon;Z,\Lambda)$, $l(q,\varepsilon;Z,\Lambda)$, $p(q,\varepsilon;Z,\Lambda)$, $y(q,\varepsilon;Z,\Lambda)$, $q'(q,\varepsilon;Z,\Lambda)$, $x(q,\varepsilon;Z,\Lambda)$, $S(q,\varepsilon;Z,\Lambda)$, and $\pi(q,\varepsilon;Z,\Lambda)$ solve the firm's optimization problem, with* $\pi(q,\varepsilon;Z,\Lambda) = \frac{p(q,\varepsilon;Z,\Lambda)}{P(Z,\Lambda)} S(p(q,\varepsilon;Z,\Lambda),\varepsilon) - w(Z,\Lambda) l(q,\varepsilon;Z,\Lambda) - \psi_x \frac{(q - S(p(q,\varepsilon;Z,\Lambda),\varepsilon))^2}{2}$.

2. $S(Z,\Lambda) = \left(\int \varepsilon^{\frac{1}{\gamma}} S(q,\varepsilon;Z,\Lambda)^{\frac{\gamma-1}{\gamma}} d\varepsilon\, d\Lambda(q\times\varepsilon)\right)^{\frac{\gamma}{\gamma-1}}$.

3. *The aggregate individual firm's profit is* $\Theta(Z,\Lambda) = \int \pi(q,\varepsilon;Z,\Lambda) d\varepsilon\, d\Lambda(q\times\varepsilon)$.

4. *The aggregate sectoral profits are given by* $\Omega(Z,\Lambda) = \left(1 - \xi(Z,\Lambda) - \frac{\kappa}{2}(\Pi(Z,\Lambda)-1)^2\right) S(Z,\Lambda)$.

5. *Goods markets clear at all dates:*
$C(Z,\Lambda) = \left(1 - \frac{\kappa}{2}(\Pi(Z,\Lambda)-1)^2\right) S(Z,\Lambda) - \frac{\psi_x}{2}\int x(q,\varepsilon;Z,\Lambda)^2\, d\varepsilon\, d\Lambda(q\times\varepsilon)$.

6. *Given* $w(Z,\Lambda)$*, the functions* $C(Z,\Lambda)$ *and* $L(Z,\Lambda)$ *solve the household optimization problem:*
$L(Z,\Lambda) = \left(\frac{w(Z,\Lambda)}{\chi C(Z,\Lambda)}\right)^{\phi}$.

7. *Labor markets clear at all dates:* $\int l(q,\varepsilon;Z,\Lambda)\, d\varepsilon\, d\Lambda(q\times\varepsilon) = L(Z,\Lambda)$*, i.e.,* $w(Z,\Lambda)$ *satisfies*
$\int l(q,\varepsilon;Z,\Lambda)\, d\varepsilon\, d\Lambda(q\times\varepsilon) = \left(\frac{w(Z,\Lambda)}{\chi C(Z,\Lambda)}\right)^{\phi}$.

8. $X(Z,\Lambda) = \int x(q,\varepsilon;Z,\Lambda)\, d\varepsilon\, d\Lambda(q\times\varepsilon)$ *and* $Q(Z,\Lambda) = \int q'(q,\varepsilon;Z,\Lambda)\, d\varepsilon\, d\Lambda(q\times\varepsilon)$.

9. *The inflation path* $\Pi(Z,\Lambda)$ *is consistent with the Phillips curve.*

10. *The law of motion* $\Gamma(\Lambda,Z)$ *is consistent with the firms' optimal decisions, such that the distribution satisfies:* $\Lambda' = \Gamma(\Lambda,Z)$.

5 Simulating the Model

To determine the economy's reaction to monetary and business cycle shocks, we first need to establish the stationary competitive equilibrium for the model. In Appendix H, we discuss the algorithm used to solve for the steady state and present the results of the equilibrium conditions in the steady state. The approach involves simulating the full model (i.e., without aggregate and monetary shocks) with a steady-state inflation rate of $\Pi_t = 1$.

Solving a heterogeneous agents model with aggregate shocks is computationally challenging. In such cases, the vector of state variables, which includes the distribution of firms, is of infinite dimension. Consequently, the policy functions depend on this infinite-dimensional object. Specifically, the vector of state variables (Z,Λ) is infinite, with Λ being the infinite component.

To address this challenge, we use the projection and perturbation methods proposed by Reiter (2009), combined with the method of Young (2010) to simulate a cross-section and solve for the recursive competitive equilibrium. This approach involves three steps: First, we discretize the model. Second, we solve for the non-stochastic steady state of the model with idiosyncratic demand shocks but no aggregate uncertainty. Finally, we linearize around this non-stochastic steady state and solve the dynamics using a rational expectations solver. Details of this algorithm can be found in Appendix H.

5.1 The calibration and steady-state distribution

Below, we detail our calibration strategy.

Preference Parameters. The discount factor is set to $\beta = 0.95$. The period utility is specified as

$$U(\cdot) = \log(C) + \frac{1}{\chi}\frac{L^{\frac{1}{\phi}+1}}{\frac{1}{\varphi}+1},$$

where the Frisch elasticity of labor supply is set to $\phi = 0.5$, as recommended by Chetty et al. (2011) for the intensive margin. We set the labor-scaling parameter to $\chi = 0.8$, which normalizes the aggregate labor supply to 0.33.

Technology and TFP Shock. The production function is assumed to be of the form $y = Zl^{\alpha}$, where α is set to 0.4. Firms face a quadratic cost for carrying inventories, with a scaling parameter set to $\psi_x = 0.5$. The TFP shock process follows a standard AR(1) process: $Z_t = Z_0 e^{z_t}$, where z_t is defined as $z_t = \rho_z z_{t-1} + u_t^z$, with $u_t^z \sim_{\text{IID}} \mathcal{N}(0, \sigma_z^2)$. We use standard values of $\rho_z = 0.95$ and $\sigma_z = 0.31\%$ to achieve a deviation of the TFP shock z_t equal to 1% at a quarterly frequency (see Den Haan (2010)).

Idiosyncratic Demand Shocks. Idiosyncratic demand shocks follow an AR(1) process: $\varepsilon_{i,t} = \rho_{\epsilon}\varepsilon_{i,t-1} + u_{i,t}^{\varepsilon}$, where $u_{i,t}^{\varepsilon} \sim_{\text{IID}} \mathcal{N}(0, \sigma_{\varepsilon}^2)$. The calibration features an autocorrelation parameter $\rho_{\epsilon} = 0.95$ and a standard deviation $\sigma_{\varepsilon} = 0.10$. This AR(1) process is discretized using the procedure from Tauchen (1986), with 5 states.

Inventories Cost Shock. In the exercises, we also account for shocks in the cost of carrying inventories. When introducing these shocks, we assume that the cost of carrying inventories follows the process $\psi_{x,t} = \rho_{\psi_x}\psi_{x,t-1} + u_t^{\psi_x}$, where $u_t^{\psi_x} \sim_{\text{IID}} \mathcal{N}(0, \sigma_{\psi_x}^2)$. We use standard values of $\rho_{\psi_x} = 0.95$ and $\sigma_{\psi_x} = 0.31\%$ to achieve a deviation of the inventories cost shock $\psi_{x,t}$ equal to 1% at a quarterly frequency.

Monetary Parameters. The rule governing the persistent monetary policy shocks affecting the nominal interest rate is defined as $\log(\iota_t) = \rho_{\iota}\log(\iota_{t-1}) + u_t^{\iota}$, with $u_t^{\iota} \sim_{\text{IID}} \mathcal{N}(0, \sigma_{\iota}^2)$. Our goal in this part is to solve the model to study the effect of a 1% contractionary monetary policy shock on the dynamics of the economy using a standard Taylor rule. To implement this exercise, we set $\rho_{\iota} = 0.5$ and $\phi_{\Pi} = 1.5$. These are standard values in the literature (see Galí (2015)). We assume an elasticity of substitution of $\gamma = 2$ and a price adjustment cost of $\kappa = 0.2$.

Table 4 provides a summary of the model parameters.

Parameter	**Description**	**Value**
Preference parameters		
β	Discount factor	0.95
φ	Frisch elasticity	0.5
χ	Scaling parameter labor supply	0.8
Technology parameters		
α	Labor share	0.4
ψ_x	Cost to carry inventories	0.5
Shock process		
ρ^z	Autocorrelation TFP shock	0.95
σ^z	Standard deviation TFP shock	0.31
ρ^ε	Autocorrelation demand shock	0.95
σ^ε	Standard deviation demand shock	0.10
ρ^{ψ_x}	Autocorrelation carry inventories shock	0.95
σ^{ψ_x}	Standard deviation carry inventories shock	0.31
ρ^β	Autocorrelation preference shock	0.95
σ^β	Standard deviation preference shock	0.31
ρ^ι	Autocorrelation monetary policy shock	0.5
σ^ι	Standard deviation monetary policy shock	0.01
Monetary parameters		
γ	Elasticity of demand with respect to price	2.0
ϕ_Π	Taylor coefficient baseline	1.5
κ	Price adjustment cost	0.2

Table 4: Parameter values in the baseline calibration. See text for descriptions and targets.

5.2 Model for Inflation Dynamics

Using the calibration from Table 4, we now solve for the model's dynamics under the assumption of a persistent shock to the nominal interest rate. Specifically, we consider an increasing nominal interest rate, with the trajectory of this rate anticipated by the agents. The results and the algorithm for solving the steady state are discussed in Appendix H.

We begin by simulating the model following two shocks: a positive shock that increases the cost of carrying inventories and a negative TFP shock. The Impulse Response Functions (IRFs) are shown in Figures 7 and 8. The goal is to determine if rising inventory costs can be rationalized by changes in TFP productivity.

Comparing these scenarios sheds light on the transmission mechanisms between inventory costs and TFP shocks. Each figure panel displays the percentage change in key variables, except for the inflation rate, which is reported in absolute terms. For instance, Panel 1 of Figure 7 shows a persistent increase in ψ_x over 100 periods, following an initial increase of 0.3%, with inflation as the exception.

Following the increase in the cost of carrying inventories, as depicted in Figure 7, firms respond by sharply reducing the total inventory levels they hold.

As inventory costs rise, firms seek to minimize their exposure to these higher costs by lowering their inventory holdings. This reduction in inventories leads to several important adjustments in the economy. Over time, as firms reduce prices to clear inventories (see Panel 3 of Figure 7), they begin to depend more on production to meet demand. This drives firms to ramp up production (Panel 6 of Figure 7), creating a stronger need for labor. As a result, labor increases (Panel 5 of Figure 7) as firms shift their focus from holding inventories to boosting production, which leads to an immediate increase in wages (Panel 4 of Figure 7).

With the need for more production, both the demand for labor and wages increase. Higher wages, in turn, further stimulate labor supply. Firms, no longer able to rely on maintaining large inventories, are forced to meet demand through real-time production, amplifying the importance of labor in the economy.

As production increases and inventories shrink, the economy shifts toward a more production-intensive equilibrium. Firms rely less on the costly storage of goods and more on their ability to quickly produce and sell. This shift reveals a critical relationship between inventory costs and broader economic performance—inventory management decisions can directly influence labor markets, price levels, and overall production decisions.[20]

Note, however, that a negative TFP shock initially leads to an increase in inventories. Following such a shock, firms tend to hold more inventories, likely in an attempt to raise prices (Panel 3 of Figure 8) to compensate for the reduced quantity of goods available for sale, as shown in Panels 6 and 7 of Figure 8. As prices rise, real wages decline (Panel 4 of Figure 8) because workers'

[20] In Appendix I.1, we illustrate a preference shock in β, following a similar approach to that used in Section 3. In this instance, the main conclusions align with those of a shock to inventory carrying costs, though the effects are more pronounced.

purchasing power erodes with the higher cost of goods. Labor supply increases (Panel 5 of Figure 8) in response to this shock, which can be interpreted as consumers offering more labor. However, this increase in labor supply is not enough to offset the reduction in production caused by the negative TFP shock.

This rise in inventory holdings occurs as firms anticipate reduced production capacity. Over time, as the negative TFP shock persists and firms experience prolonged reductions in production capacity, the inventory levels are no longer sustainable. The decrease in production eventually forces firms to rely on these inventories to meet ongoing demand, leading to the reduction of stockpiles.

As can be seen in Panel 2 of Figure 8 over time we can see a reduction in inventories. This reduction can be attributed to several factors. First, firms are unable to replenish their inventories as they are producing less. As a result, they are forced to deplete their existing reserves in order to meet ongoing demand. Second, holding large inventories incurs costs, such as storage represented by ψ_x in our model. With reduced productivity, these costs become more burdensome, pushing firms to reduce their inventory holdings. Finally, while firms may initially raise prices to take advantage of the scarcity created by lower production, this strategy is not sustainable in the long term. As demand weakens due to higher prices, the incentive to hold onto inventories decreases, leading firms to sell off their stock at lower prices, further contributing to the reduction in inventory levels.

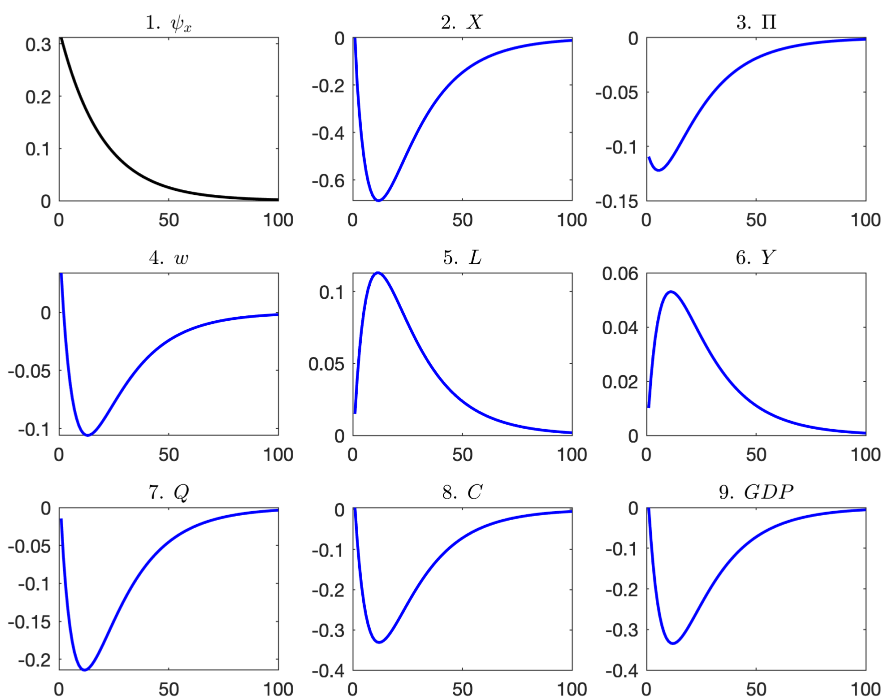


Figure 7: Impulse Response Functions for the selected variables following a positive shock to ψ_x.

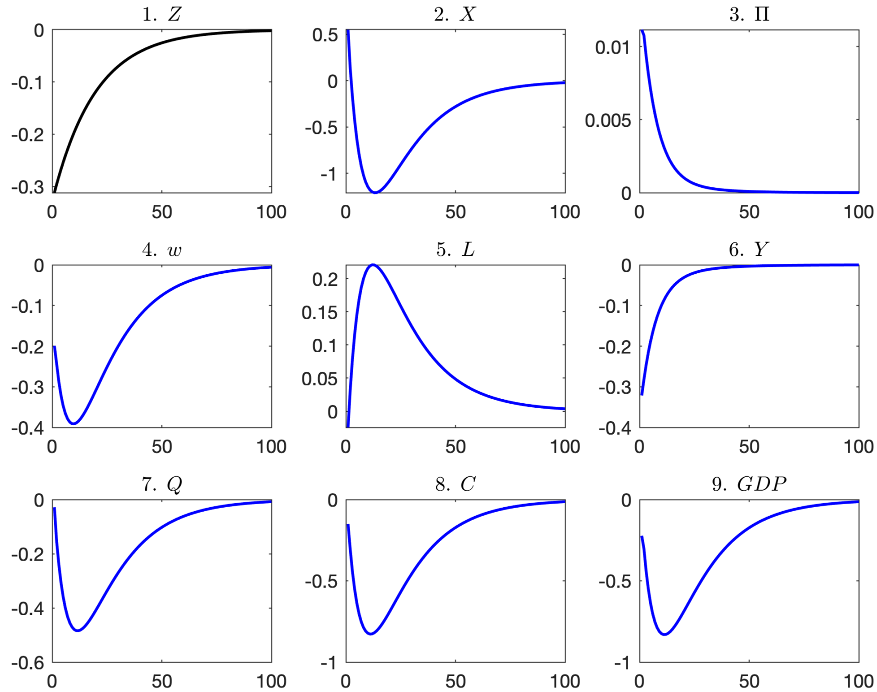


Figure 8: Impulse Response Functions for the selected variables following a negative shock to Z.

The dynamics of increasing inventory carrying costs and a negative TFP shock are comparable in how firms adjust inventory levels. When carrying costs rise, firms reduce inventories to avoid higher expenses, relying more on production to meet demand, as shown in Figure 7. Similarly, after a negative TFP shock, firms initially increase inventories to buffer against lower output but eventually deplete stockpiles as production remains low (Figure 8). In both cases, rising costs or reduced productivity drive firms to reduce inventories, impacting prices, labor demand, and overall equilibrium.

Comparing the two scenarios, we observe that the volatility of the main variables is greater for the same shock magnitude when experiencing a TFP shock compared to a shock affecting the cost of carrying inventories. This is likely because TFP shocks pose a deeper challenge by disrupting the fundamental productive capacity of the economy, thereby amplifying volatility across key variables.

These findings are confirmed by the second-order moments presented in Table 5, reported in columns (1) and (2).

		ψ_x Int. rate shock	Z Int. rate shock	ι $\phi_\Pi = 1.5$	ι Int. rate
w	Mean	1.002	1.008	1.003	1.012
	Std	0.005	0.018	0.021	0.045
Π	Mean	1.002	1.000	1.001	1.007
	Std	0.537	0.025	0.689	2.776
X	Mean	0.321	0.324	0.322	0.336
	Std	0.031	0.055	0.115	0.239
C	Mean	1.592	1.607	1.596	1.630
	Std	0.015	0.038	0.057	0.117
L	Mean	0.261	0.261	0.261	0.259
	Std	0.005	0.010	0.018	0.036
Q	Mean	0.879	0.883	0.880	0.893
	Std	0.010	0.022	0.035	0.076
Y	Mean	0.558	0.560	0.558	0.556
	Std	0.002	0.007	0.008	0.016
GDP	Mean	1.626	1.642	1.630	1.666
	Std	0.015	0.038	0.057	0.119
		Correlations			
corr(X, Π)		0.9408	-0.3931	0.8702	0.8993
corr(X, X_{-1})		0.9946	0.9869	0.7907	0.9818
corr(GDP, GDP_{-1})		0.9950	0.9950	0.8213	0.9827

Table 5: First and second moments for key variables are provided for the following scenarios: a shock to ψ_x, a shock to Z, a shock to ι with a Taylor rule ($\phi_\Pi = 1.5$), and a shock to ι with a known interest rate path.

For each variable, we report the steady-state value (labeled "Mean") and the normalized standard deviation in percentage terms, which is the standard deviation divided by the mean (labeled "Std"), except for inflation, where only the standard deviation is reported. The second part of the table presents correlations. Notice that the results in Table 5 are consistent with the IRFs for the aggregate variables.

To conclude this section, we conduct an exercise comparing two scenarios: one with an initial quantity of inventories and another with a higher initial inventory level. The objective of this comparison is to analyze how a shock to the carrying cost of inventories affects the evolution of total inventories and its impact on prices. We observe that a higher initial level of inventories generally results in greater volatility of the selected variables.

Furthermore, for the same shock, the reduction in total inventories is more pronounced when starting with a higher inventory level. We hypothesize that the decrease in prices will be more substantial in the economy with a higher initial inventory level. This is because firms have greater incentives to reduce their inventories and may achieve this by lowering prices further to boost demand. Our analysis of the IRFs confirms this hypothesis: a higher initial inventory level leads to a greater reduction in prices for the same shock. The IRFs are displayed in Figure 9.

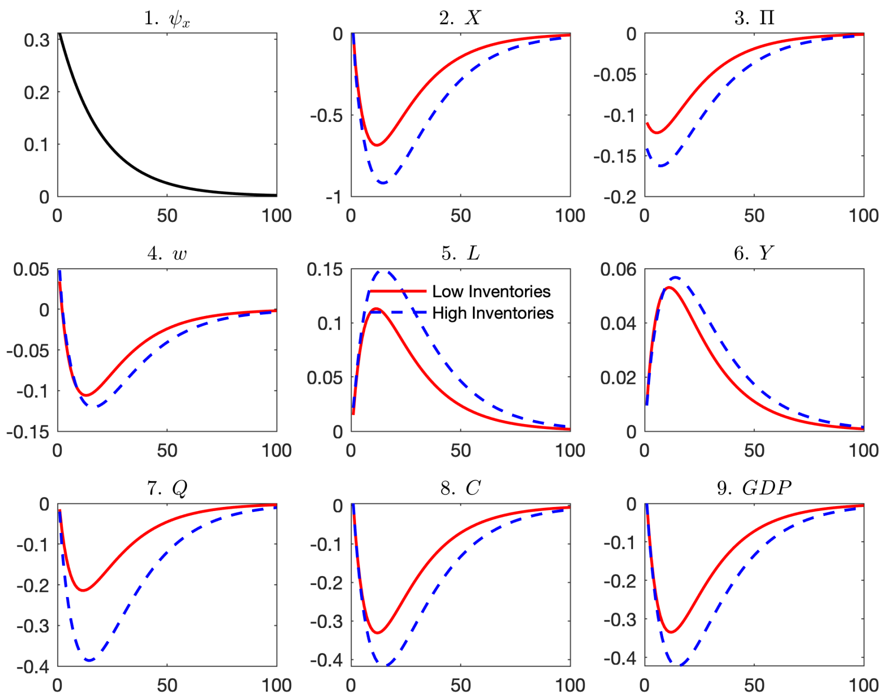


Figure 9: Impulse Response Functions for the selected variables following a positive shock to ψ_x, comparing two scenarios: a higher inventory level and a lower inventory level.

Doing a similar exercise for a negative TFP we can notice that the higher the inventoriers level, the lower will be the increase in prices. These resutls are depicted in Figure 10.

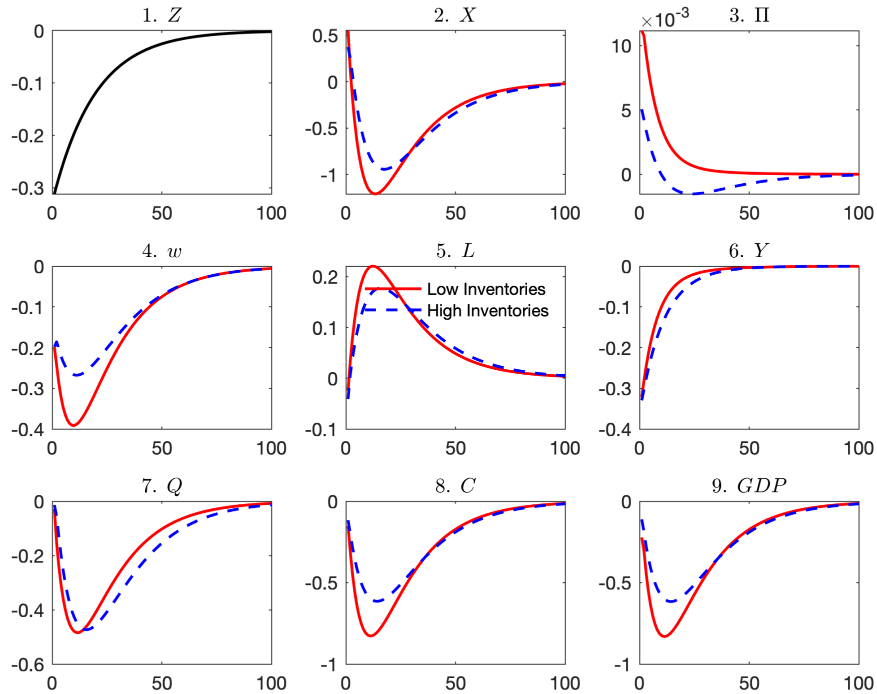


Figure 10: Impulse Response Functions for the selected variables following a negative shock to Z, comparing two scenarios: a higher inventory level and a lower inventory level.

The goal of this exercise, as stated in the simple model presented in Section 3, is to demonstrate that inventories play a crucial role in price decisions. Specifically, we claim that a higher inventory level leads to a smaller reduction in prices for the same type of shock.

5.3 Monetary Policy

We now examine an economy where monetary policy is implemented via a Taylor rule as in

$$\frac{R_t^N}{R^N} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}, \tag{66}$$

where R^N is the steady-state nominal interest rate, Π is the steady-state inflation rate, and ϕ_Π is the response of the nominal interest rate to inflation.

First, we compare two scenarios: one where the Central Bank adopts a policy rule according to the Taylor Rule with $\phi_\Pi = 1.5$ as in Galí (2015) and reacts to deviations from the steady-state inflation rate set at $\Pi_t = 1$, and another where there is a pesisent shock to the nominal interest rate as discussed in Section 5.2.

The goal is to assess the effects of the endogenous response of monetary policy and to understand the model dynamics under different inflation paths. When the Taylor Rule is applied, as expected, all selected variables exhibit lower volatility and return more quickly to their steady-state values. In particular, the Taylor Rule helps stabilize inflation, preventing it from becoming excessively volatile.

Following a contractionary monetary policy shock, the level of inventories decreases, but the reduction in prices is less pronounced compared to the scenario where agents can anticipate the path of interest rate. This outcome highlights the Taylor Rule's role in minimizing deviations from steady-state values, thereby moderating the impact on prices.

Those results are depicted in Figure 11 for a 1% contractionary monetary policy.

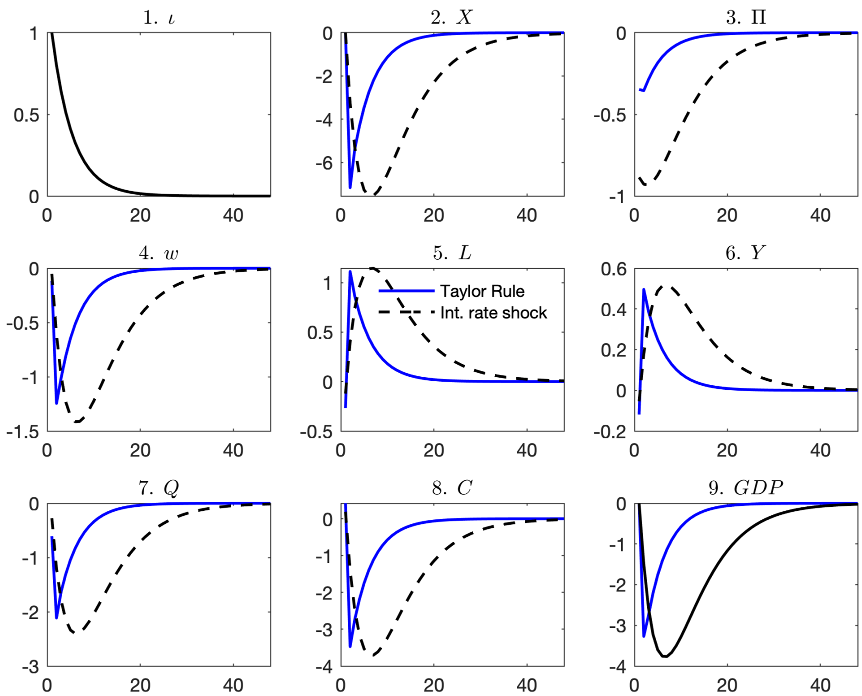


Figure 11: Impulse Response Functions for selected variables following a 1% contractionary monetary policy.

As previously argued, an increase in the carrying cost of inventories can be linked to a contractionary monetary policy. In Appendix J, we analyze shocks on ψ_x and Z when the monetary authority follows a Taylor rule. Our analysis demonstrates that the effects of a monetary policy contraction are equivalent to those of an increase in inventory carrying costs. Specifically, by comparing Figures 11 and 48 in Appendix J, we observe that the effects are strikingly similar.

These findings are confirmed by the second-order moments presented in Table 5, reported in columns (3) and (4).

Finally, we compare two distinct situations: one with a low level of inventories and another with a higher level. First, we examine a monetary policy contraction when the path of interest rate is known. In this scenario, we observe that a higher inventory level leads to a greater reduction in the total inventory level. Consequently, the reduction in prices needed to decrease these inventory levels is more pronounced. As discussed in Section 3.5, the volatility of the variables is higher in the economy with a larger inventory level, likely due to a stronger reaction in this case. These results are depicted in Figure 12.

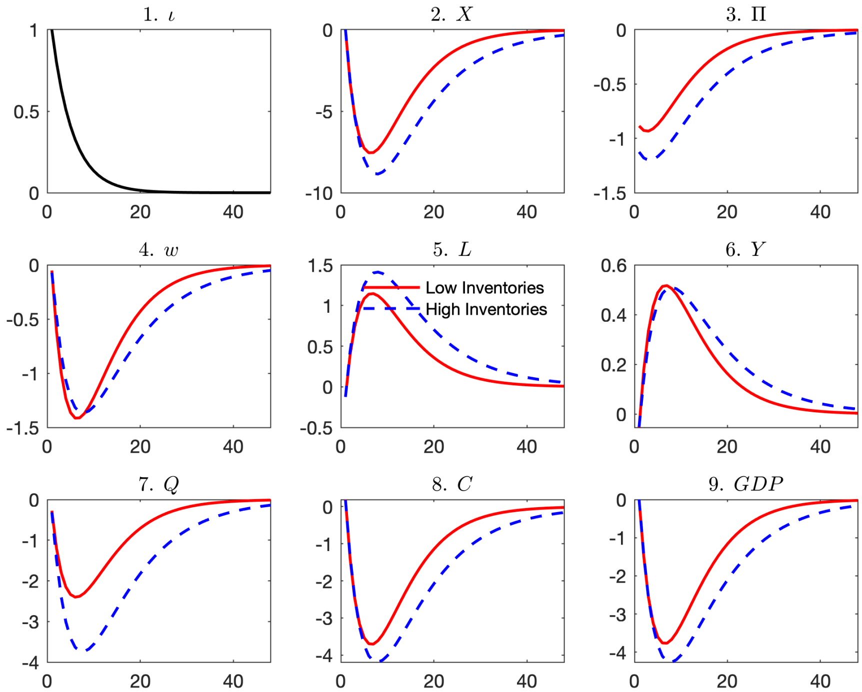


Figure 12: Impulse Response Functions for selected variables following a 1% contractionary monetary policy without a Taylor Rule, compared for two distinct levels of inventories.

Now we present the results when the Taylor rule is applied with a coefficient of $\phi_{\Pi} = 1.5$. We observe that, under the Taylor rule, the decrease in prices is similar across both inventory levels. This outcome occurs because the Taylor rule aims to minimize deviations from the steady-state inflation rate. Consequently, the paths of the selected variables are similar in both cases, as the Taylor rule effectively moderates the impact of monetary policy shocks. Figure 13 shows the IRFs.

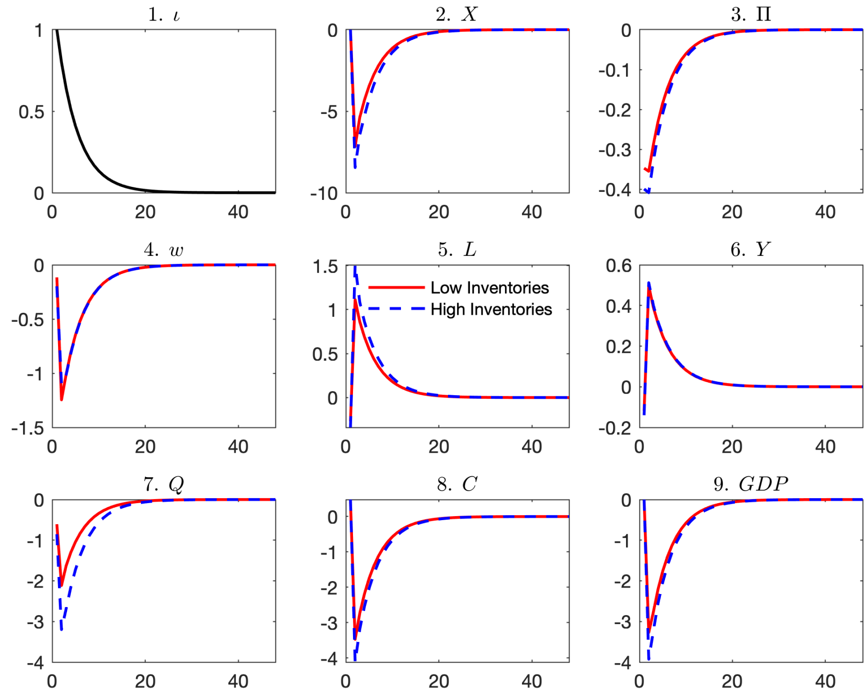


Figure 13: Impulse Response Functions for selected variables following a 1% contractionary monetary policy with a Taylor Rule of $\phi_\Pi = 1.5$, compared for two distinct levels of inventories.

In order to understand the model dynamics when inflation follows a different path, we examine two inventory levels, higher and lower, in a scenario where the Taylor rule coefficient is set to $\phi_\Pi = 1.01$. This represents a situation where the endogenous response of monetary policy is weaker, and thus, inventories should play a stronger role in the inflation path. The IRFs for this case are shown in Figure 14. By comparing Panel 3 of Figure 14 with Panel 3 of Figure 13, we observe that a lower Taylor coefficient results in a greater divergence between the two inflation paths. This indicates that the less hawkish the Central Bank, the more inventories influence the inflation trajectory. Moreover, one can observe that the overall effect in reducing inflation is now stronger, which confirms our claim that inventories play now a more significant role. Consequently, monetary policy contractions will have a larger effect in reducing inflation as the inventory level increases.

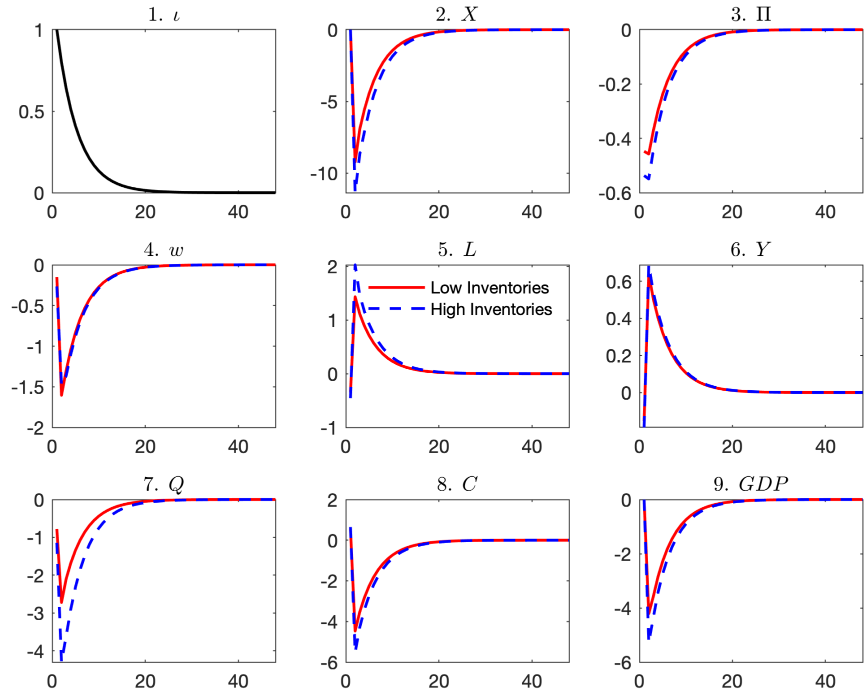


Figure 14: Impulse Response Functions for selected variables following a 1% contractionary monetary policy with a Taylor Rule of $\phi_{\Pi} = 1.01$, compared for two distinct levels of inventories.

This occurs because the model already incorporates a natural mechanism through firms' price-setting decisions that helps reduce inflation. This result suggests that monetary contractions reduce inflation more effectively when inventory levels are high, implying that central banks may not need to pursue overly aggressive tightening when inventory levels are elevated.[21]

5.4 Effect of Monetary Policy Shocks on Inflation

Although the previous exercise illustrates the role of inventories in the inflation path, particularly when the monetary authority is less hawkish, it presents a challenge. While we compared overall inventory levels that are relatively close, with one scenario featuring a higher total quantity, the steady states are not the same. This suggests that other factors are likely at play.

To better understand the impact of monetary policy shocks on inflation in a high-inventory context and to address this inconsistency, we will conduct the following exercise. We introduce a shock to the variance of the firm's idiosyncratic shocks, which results in an increase in inventory levels due to greater firm-level risk stemming from the uncertainty shock. This approach allows us to compare economies operating under the same steady state.

Next, we introduce a contractionary monetary policy shock of 1% as described in equation (57) at two different points in time: when inventory levels are high and when they are low. By simulating these scenarios, we can observe how the inflation paths differ between the two cases, highlighting the effects of monetary policy shocks when inventory levels are elevated.

[21]In Appendix K we show the same exercise for the case where we have a negative shock in Z.

Figure 15 illustrates the results of this exercise.

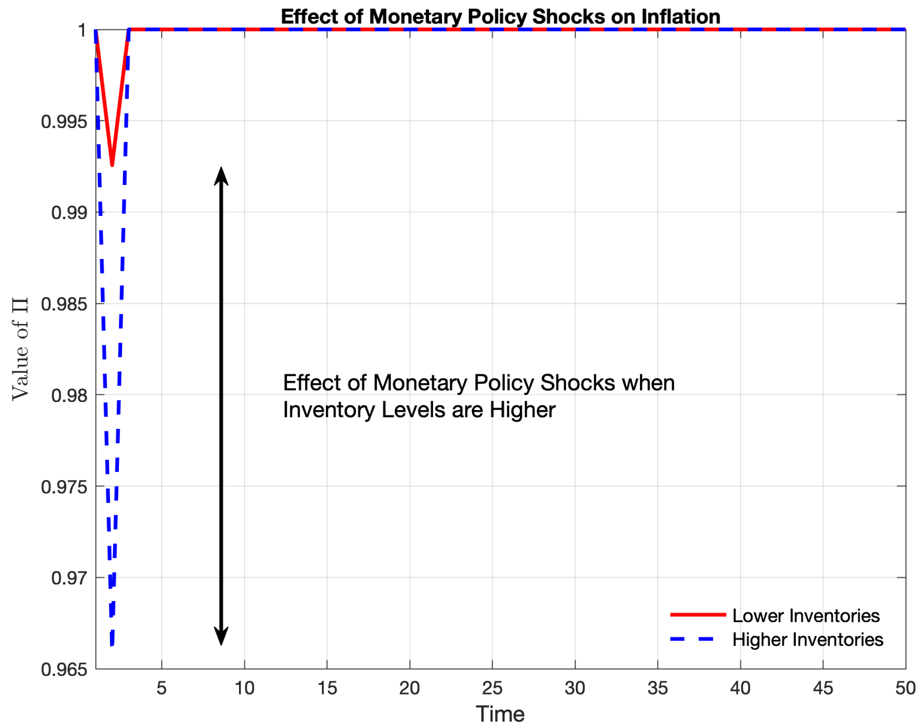


Figure 15: Effect of Monetary Policy Shock on Inflation with $\phi_{\Pi} = 1.5$, compared for two distinct levels of inventories with the same steady state.

Unlike the previous exercises we have conducted, the objective here is to examine how variations in inventory levels due to an uncertainty shock influence price dynamics. Since we are comparing scenarios that maintain the same steady state but differ in inventory levels, it is evident that the differences in inventories are driving the results.

By introducing an uncertainty shock that increases the variance of the uncertainty shock by 1%, we can observe that a monetary policy contraction of the same magnitude results in approximately a 3 percentage point decrease in inflation. These results highlight the significance of inventory levels throughout the business cycle in understanding their impact on inflation and emphasize that higher inventory levels lead to a greater reduction in prices following a contractionary monetary policy shock.

6 Empirical evidence from the US housing market

Our model also gives rise to a clear, testable hypothesis: prices should be more responsive to the stance of monetary policy when inventories stand at a higher level.

We test this prediction in the housing market, for two main reasons. Firstly, by looking at housing costs, we can directly get at a major component of the consumption basket (recall footnote 3). Secondly, in the housing market, the concepts residing at the core of our theory are relatively well-defined. Ideally, one would have firm-level data on inventories *at the goods level* while being able to match those with associated goods-level prices at a high frequency. In practice, however, price- and inventory data sum over various types of goods, which creates major challenges for our exercise. By limiting our focus to housing, we are able to side-step these aggregation issues, while at the same time looking at a significant item in the consumption basket. We do acknowledge that our strategy implies that our findings may be specific to the housing market. But even in that case, we see our results as interesting given the importance of housing in the typical consumption basket.

To test our model's prediction, and thereby the cost-of-carry channel of monetary policy transmission, we run Local Projections (LPs) of the following form, at the monthly frequency:

$$\Delta^h \ln HC_{t+h} = \alpha_h + \beta_h MPS_t + \gamma_h \left(MPS_t \times INV_t \right) + \delta_h X_t + \epsilon_{t,h}, \tag{67}$$

where $\Delta^h \ln HC_{t+h} \equiv \ln HC_{t+h} - \ln HC_{t-1}$ is the cumulative change in the natural log of the housing cost series "HC" over h months. To measure the cost of housing, we use the CPI owner's equivalent rent (OER) series (but results are robust to using the housing-component in the PCE series instead, or to using the "shelter" component of the CPI, which is slightly broader than OER).[22] The variable "INV" represents the fraction of homes being vacant, which represents the inventory of homes looking to be utilized (see Appendix L.1 for details on how this variable is constructed). Finally, "MPS" is the monetary policy shock, which we draw from the series provided by Bauer & Swanson (2023). Since they take great care in ensuring that the series is orthogonalized with respect to available data, we run the LPs by only controlling (in X_t) for lags of the shock and its interactions with the housing inventory indicator (so that the righthand-side of (67) features 12 months worth of MPS-realizations and $MPS \times INV$-interactions).[23] As documented in Appendix L.2, the Bauer-Swanson shocks produce very intuitive responses in standard variables, like consumer prices, industrial production, unemployment, and equity prices – comforting us that the series is getting at true monetary policy shocks.

When estimating a regression of the form of (67) *without* the interaction term,

$$\Delta^h \ln HC_{t+h} = \alpha_h + \beta_h MPS_t + \delta_h X_t + \epsilon_{t,h}, \tag{68}$$

we see in Figure 16 that contractionary monetary policy shocks tend to lower housing costs. Figure 16 points to a substantial lag in the response, which is to be expected given the construction of the OER series (which not only looks at rentals offered on the market contemporaneously, but takes into account that rents only tend to change when leases expire; see Conner et al. (2024) and Cotton (2024) for more details on the calculation of OER).

[22] Relative to OER, the CPI-shelter series also includes "lodging away from home" and insurance costs, among other items.

[23] Note that the Frisch–Waugh–Lovell theorem implies that this is equivalent to a regression in which MPS represents the *un*orthogonalized shock but the vector of controls X_t includes all variables that Bauer & Swanson (2023) orthogonalize on (Lloyd & Manuel 2024).

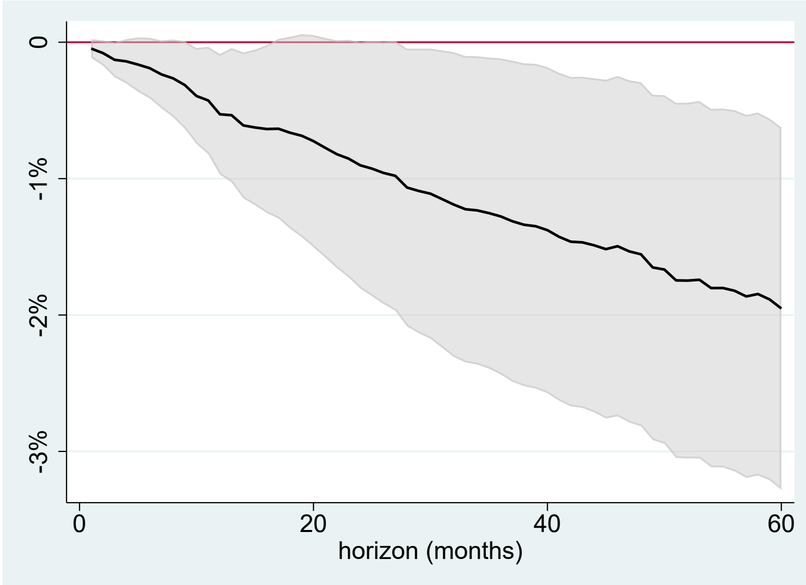


Figure 16: Response of CPI OER to a 25-bp contractionary monetary policy shock, estimated via equation (68). Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

This result, however, is unconditional in nature – whereas our model suggests that inventory levels matter for the transmission of monetary policy. This motivates us to estimate (67), which includes an interaction term "$MPS_t \times INV_t$" between the monetary policy shock and our "housing inventory" variable (the fraction of homes being vacant). Results of this exercise are plotted in the next two figures. Figure 17 shows that, when the housing vacancy rate stands at its sample average, a monetary tightening has virtually no effect on the cost of housing. Looking at the coefficient "γ" on the interaction term (in Figure 18) however demonstrates that monetary policy has greater leverage over the cost of housing when more properties are vacant – i.e., when there is a greater inventory of housing looking for an occupant.[24] As shown in Appendix L.3, this result is highly robust to adding further controls, including the rate of unemployment (to proxy for the state of the business cycle) which suggests that we are not picking up variations driven by business cycle fluctuations.[25] This supports the prediction from our theoretical model, that inventories matter for the transmission of monetary policy – with higher inventory levels making sellers more responsive to changes in the stance of monetary policy. This is consistent with the notion that a tighter housing market (lower INV_t) enables landlords to pass on any increases in their borrowing costs (e.g., following a monetary policy tightening). However, when there are more vacant properties (higher INV_t), landlords have less market power and they become more inclined to lower their price in response to a monetary contraction – reflecting the higher opportunity cost of not having the property occupied. As Figure 62 in Appendix L.3 shows, our main result is also visible when using the overall CPI as dependent variable in (67) – which is perhaps to be expected given that OER accounts for about one-third of the total CPI basket. A similar picture emerges when looking at the overall PCE index.

[24] Interestingly, Robert Reffkin (the founder and CEO of real estate firm Compass) conveyed exactly this notion when stating in 2023 that real estate prices "have peaked, and that rising inventories could bend prices in the months ahead" (see https://uk.marketscreener.com/quote/stock/COMPASS-INC-120835016/news/Robert-Reffkin-CEO-of-Compass-Real-estate-prices-down-inventories-up-45444716/).

[25] The correlation between the housing vacancy rate ("INV_t") and the rate of unemployment is quite low, at 0.24.

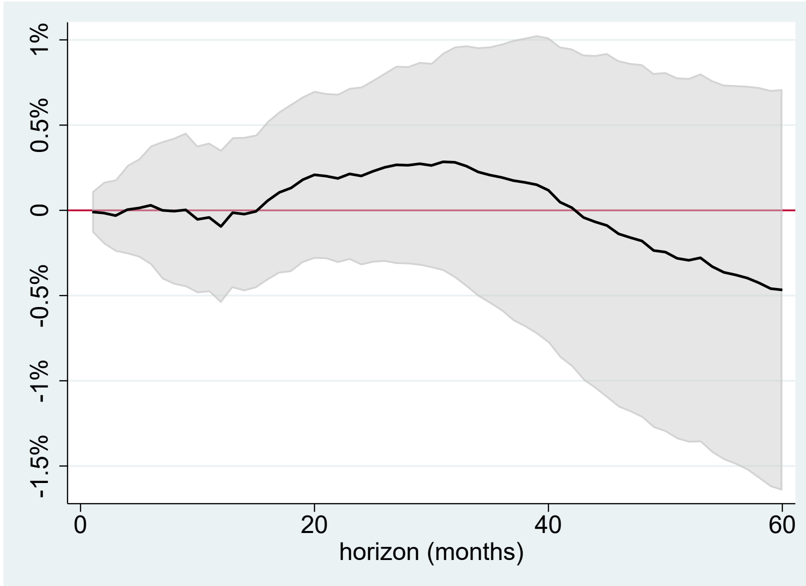


Figure 17: Response of CPI OER to a 25-bp contractionary monetary policy shock, estimated via equation (67), when the home vacancy rate "INV_t" stands at its historical average (INV_{avg}). The figure plots $\hat{\beta}_h + \hat{\gamma}_h \cdot INV_{avg}$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

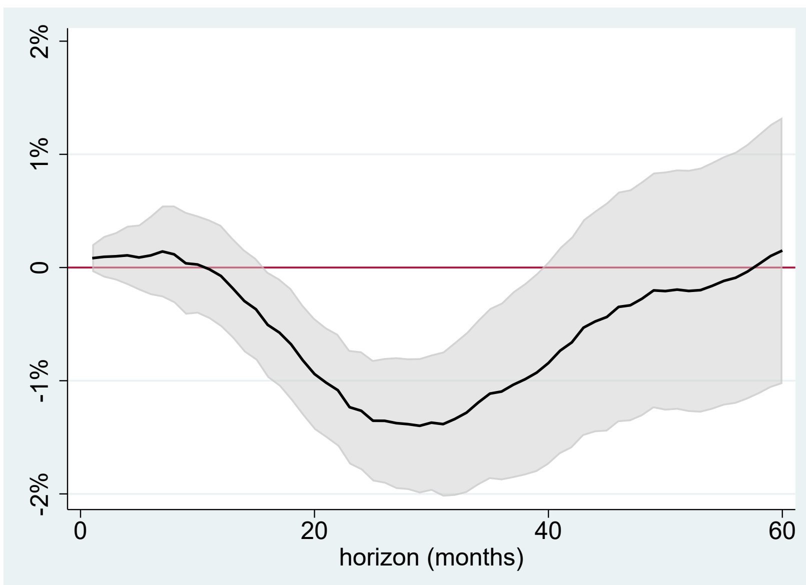
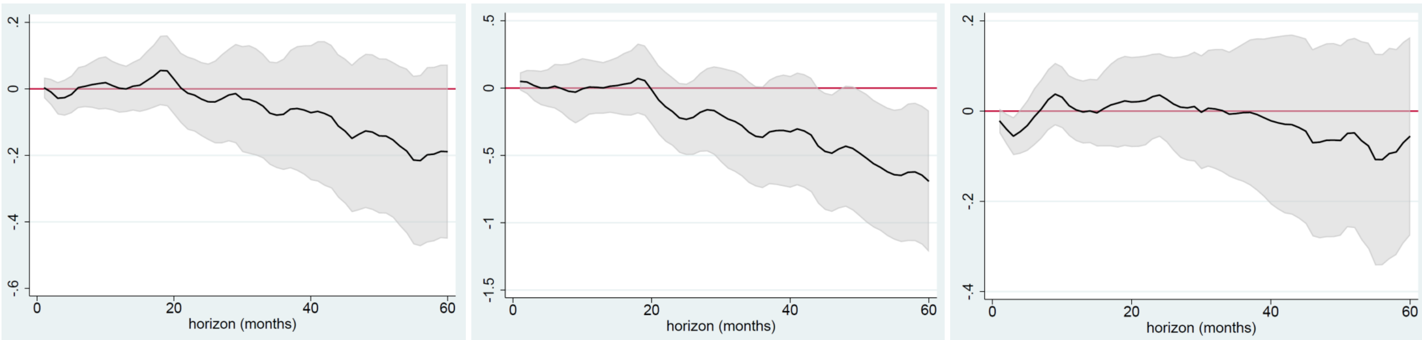


Figure 18: Additional response of CPI OER to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

Finally, the data also confirm the model's prediction regarding the inventory of housing – with the home vacancy rate ("INV_t") falling in response to a contractionary monetary policy shock; as Figure 19 shows, this result is mainly driven by rentals (as opposed to owner-occupied houses). The direction of this general finding is consistent with the cost-of-carry logic, which tells us that tighter monetary policy implies a greater opportunity cost for leaving a home unoccupied. Our result in Figure 18 suggests that, one way in which "idle time" is being reduced when the housing inventory stands at an elevated level, is by reducing the price.



(a) Housing vacancy rate (b) Rental vacancy rate (c) Homeowner vacancy rate

Figure 19: Response of (a) home vacancy rate, (b), rental vacancy rate (c) homeowner vacancy rate to a 25-bp contractionary monetary policy shock, estimated via $\Delta^h INV_{t+h} = \alpha_h + \beta_h MPS_t + \delta_h X_t + \epsilon_{t,h}$. The figure plots $\hat{\beta}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

7 Conclusion

We develop a model to analyze the cost-of-carry channel of monetary policy, focusing on how interest rates impact firms' inventory management and pricing strategies. By examining the effects of higher interest rates on the costs of holding inventories, we show that firms are incentivized to reduce prices to mitigate these costs, thereby influencing inflation dynamics.

Applying our model to the U.S. housing market, we find that higher inventory levels intensify the disinflationary impact of monetary policy. This suggests that monetary policy can be less aggressive in high-inventory environments to achieve similar inflation outcomes.

Our methodology highlights the significance of inventory management in monetary policy transmission, offering a refined perspective on inflation dynamics that complements traditional aggregate demand channels. This approach provides valuable insights for shaping effective monetary policy strategies.

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Appendix

A Proof of Proposition 1

Applying the Implicit Function Theorem to (3), we obtain:

$$\frac{\partial p}{\partial \psi_x} = \frac{x_0 - S(p)}{\psi_x S'(p) + \frac{S''(p)}{S'(p)^2} S(p) - 2}.$$

Since the non-negativity constraint on inventories implies that $S(p) \leq x_0$, we have $\frac{\partial p}{\partial \psi_x} < 0 \Leftrightarrow \psi_x S'(p) + \frac{S''(p)}{S'(p)^2} S(p) - 2 < 0$. Given that $S'(p) < 0$, this condition holds when $\frac{S''(p)}{S'(p)^2} S(p) < 2$.

From the chain rule, it follows that $S'(p) = \frac{1}{p'(S)}$ and $S''(p) = -S'(p)^3 p''(S) = -\frac{p''(S)}{p'(S)^3}$. Using these relationships, we can rewrite $\frac{S''(p)}{S'(p)^2} S(p) = -\frac{p''(S)}{p'(S)} S(p)$. This reflects the convexity of the demand curve, and Mrázová & Neary (2017) shows that profit-maximizing behavior guarantees that $-\frac{p''(S)}{p'(S)} S(p) < 2$, which implies that $\frac{S''(p)}{S'(p)^2} S(p) - 2 < 0$, thereby proving that $\frac{\partial p}{\partial \psi_x} < 0$. The second part of the proposition follows trivially from the observation that the steepness of this derivative increases with x_0.

B Equilibrium conditions

B.1 The firm's first-order condition

By taking first-order conditions of the problem in (13) - (15) with respect to p and y, we obtain the following results, respectively:

$$\frac{S(p) + pS'(p)}{S'(p)} + \psi_x(q - S(p)) = \beta \mathbb{E}[V'(q')] + \mu,$$
$$\psi_y y = \beta \mathbb{E}[V'(q')].$$

Notice that $V'(q) = -\psi_x(q - S(p)) + \beta \mathbb{E}[V'(q')] + \mu$. Therefore, using the envelope condition and the aforementioned first-order conditions, we can summarize the solution to this problem by the following equations:

$$\frac{S(p) + pS'(p)}{S'(p)} + \psi_x(q - S(p)) = \psi_y y + \mu, \tag{69}$$

$$\psi_y y = \beta \mathbb{E}\left[-\psi_x(q' - S(p')) + \psi_y y' + \mu'\right], \tag{70}$$

$$q' = q - S(p) + y, \tag{71}$$

$$\mu(q - S(p)) = 0. \tag{72}$$

Observe the elasticity of demand is given by $E_p^S = \dfrac{pS'(p)}{S(p)}$, which implies we can rewrite $\dfrac{S(p)+pS'(p)}{S'(p)} = \dfrac{S(p)}{S'(p)}(1+E_p^S)$.

Using this expression and manipulating equations (16) - (19) we have:

$$(\psi_y + \beta\psi_x)y = \beta\frac{S(p)}{S'(p)}(1+E_p^S) + \beta\psi_x\mathbb{E}(S(p')) + \beta\psi_y\left(\mathbb{E}(y') - y\right) + \beta\left(\mathbb{E}(\mu') - \mu\right). \tag{73}$$

B.2 Proof of Proposition 2

Applying (17) for the case when the economy was in state H and is currently in state L, leads to:

$$\psi_y y_{LH} = \beta\left\{(-\psi_x(q_{LH} - S(p_{LH}) + y_{LH} - S(p_{HL})) + \psi_y y_{HL} + \mu_{HL})\right\}. \tag{74}$$

Now, applying (16) and observing:

$$\frac{S(p_{LH})}{S'(p_{LH})}(1+E^S_{p_{LH}}) + \psi_x(q_{LH} - S(p_{LH})) = \psi_y y_{LH} + \mu_{LH}. \tag{75}$$

Observe that if the economy transitions to the state L, then we have $S(p_{LH}) < q_{LH}$. Therefore, the Lagrange multiplier in this case will be:

$$\mu_{LH} = 0. \tag{76}$$

Replacing (75) into (74) we finally reach:

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_{LH} = \beta\frac{S(p_{LH})}{S'(p_{LH})}(1+E^S_{p_{LH}}) + \beta\psi_x S(p_{HL}) + \beta\psi_y y_{HL} + \beta\mu_{HL}. \tag{77}$$

Using (17) for the case when the economy was in state L and is currently in state H:

$$\begin{aligned}\psi_y y_{HL} = \beta\Big\{&\alpha(-\psi_x(q_{HL} - S(p_{HL}) + y_{HL} - S(p_{HH})) + \psi_y y_{HH} + \mu_{HH}) + \\ &(1-\alpha)(-\psi_x(q_{HL} - S(p_{HL}) + y_{HL} - S(p_{LH})) + \psi_y y_{LH} + \mu_{LH})\Big\}.\end{aligned} \tag{78}$$

Using (16) for this case:

$$\frac{S(p_{HL})}{S'(p_{HL})}(1+E^S_{p_{HL}}) + \psi_x(q_{HL} - S(p_{HL})) = \psi_y y_{HL} + \mu_{HL}. \tag{79}$$

Rearranging the terms above:

$$\begin{aligned}(\psi_y + \beta\psi_x)y_{HL} = &-\beta\psi_x(q_{HL} - S(p_{HL})) + \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH})) + \\ &\beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}.\end{aligned} \tag{80}$$

Using the same idea for the case the economy was in state H and is currently in state H we reach:

$$(\psi_y + \beta\psi_x)y_{HH} = -\beta\psi_x(q_{HH} - S(p_{HH})) + \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH}))+ \\ \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}. \tag{81}$$

Replacing (79) into (77) results in:

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_{LH} = \beta\frac{S(p_{LH})}{S'(p_{LH})}(1 + E^S_{p_{LH}}) + \beta\frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) + \beta\psi_x q_{HL}. \tag{82}$$

Here, we assume that in all cases, when the current situation of the firm reflects a High-Demand shock, the firm will have $S(p_H) \geq q_H$. Consequently, the firm will set prices such that $S(p_{HH}) = q_{HH}$ and $S(p_{HL}) = q_{HL}$. Hence, the prices when the current state is a High-Demand shock will be:

$$p_{HH} = \bar{S}^{-1}\left(\frac{q_{HH}}{1+\Delta}\right), \tag{83}$$

$$p_{HL} = \bar{S}^{-1}\left(\frac{q_{HL}}{1+\Delta}\right), \tag{84}$$

where we let the demand function $\bar{S}$ admit an inverse. Also, observe that regardless of the previous state being L or H, we argue that there exists a price at which the quantity available will be able to meet the demand. The Lagrange multiplier constraint in this case, given by equation (16), is such that:

$$\mu_{HH} = \frac{S(p_{HH})}{S'(p_{HH})}(1 + E^S_{p_{HH}}) - \psi_y y_{HH}, \tag{85}$$

$$\mu_{HL} = \frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) - \psi_y y_{HL}. \tag{86}$$

We need to identify p_{HH}, p_{HL}, p_{LH}, y_{HH}, y_{HL}, and y_{LH} as functions of q. The equations that characterize this system are given by:

$$p_{HH} = S^{-1}(q_{HH}), \tag{87}$$

$$p_{HL} = S^{-1}(q_{HL}), \tag{88}$$

$$\frac{S(p_{LH})}{S'(p_{LH})}(1 + E^S_{p_{LH}}) + \psi_x(q_{LH} - S(p_{LH})) = \psi_y y_{LH}, \tag{89}$$

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_{LH} = \beta\frac{S(p_{LH})}{S'(p_{LH})}(1 + E^S_{p_{LH}}) + \beta\frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) + \beta\psi_x q_{HL}, \tag{90}$$

$$(\psi_y + \beta\psi_x)y_{HL} = \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH})) + \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}, \tag{91}$$

$$(\psi_y + \beta\psi_x)y_{HH} = \beta\psi_x(\alpha S(p_{HH}) + (1-\alpha)S(p_{LH})) + \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH}, \tag{92}$$

$$\mu_{HH} = \frac{S(p_{HH})}{S'(p_{HH})}(1 + E^S_{p_{HH}}) - \psi_y y_{HH}, \tag{93}$$

$$\mu_{HL} = \frac{S(p_{HL})}{S'(p_{HL})}(1 + E^S_{p_{HL}}) - \psi_y y_{HL}. \tag{94}$$

B.3 Proof of Result 1

Assume the demand function is given by:

$$\bar{S}(p) = \frac{1}{p^{\gamma}}.$$

In this case, the system given by equations (21) - (28) is as follows:

$$p_{HH} = \left(\frac{1+\Delta}{q_{HH}}\right)^{\frac{1}{\gamma}}, \tag{95}$$

$$p_{HL} = \left(\frac{1+\Delta}{q_{HL}}\right)^{\frac{1}{\gamma}}, \tag{96}$$

$$p_{LH}\eta + \psi_x\left(q_{LH} - \frac{1}{p_{LH}^{\gamma}}(1-\Delta)\right) = \psi_y y_{LH}, \tag{97}$$

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_{LH} = \beta\eta\left(p_{LH} + p_{HL}\right) + \beta\psi_x q_{HL}, \tag{98}$$

$$\begin{aligned}(\psi_y + \beta\psi_x)y_{HL} = \beta\psi_x\left(\alpha\frac{1}{p_{HH}^{\gamma}}(1+\Delta) + (1-\alpha)\frac{1}{p_{LH}^{\gamma}}(1-\Delta)\right) + \\ \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH},\end{aligned} \tag{99}$$

$$\begin{aligned}(\psi_y + \beta\psi_x)y_{HH} = \beta\psi_x\left(\alpha\frac{1}{p_{HH}^{\gamma}}(1+\Delta) + (1-\alpha)\frac{1}{p_{LH}^{\gamma}}(1-\Delta)\right) + \\ \beta\psi_y(\alpha y_{HH} + (1-\alpha)y_{LH}) + \beta\alpha\mu_{HH},\end{aligned} \tag{100}$$

$$\mu_{HH} = p_{HH}\eta - \psi_y y_{HH}, \tag{101}$$

$$\mu_{HL} = p_{HL}\eta - \psi_y y_{HL}, \tag{102}$$

where $\eta \equiv \left(\frac{\gamma-1}{\gamma}\right)$.

We are looking for an equilibrium such that $x'_{HH} = x'_{HL} = 0$. Therefore, we have:

$$\begin{aligned} q'_{HH} &= y_{HH} + x'_{HH} = y_{HH}, \\ q'_{HH} &= y_{HL} + x'_{HL} = y_{HL}, \end{aligned}$$

which means $q'_{HH} = y_{HH} = y_{HL} \equiv y_H$. Also, observe that the only way to reach state L is from state H, which means:

$$\begin{aligned} q'_{LH} &= y_{HH} + x'_{HH} = y_{HH}, \\ q'_{LH} &= y_{HL} + x'_{HL} = y_{HL}, \end{aligned}$$

which means $q'_{LH} = y_{HH} = y_{HL} \equiv y_H$. This implies:

$$q'_{HH} = q'_{LH} = y_{HH} = y_{HL} \equiv y_H. \tag{103}$$

Observe now that given the previous state is L, necessarily the current state is H. Therefore:

$$q'_{HL} = y_{LH} + x'_{LH}, \tag{104}$$

where $x'_{LH} = q_{LH} - S(p_{LH}) = q_{LH} - \frac{1}{p_{LH}^{\gamma}}(1-\Delta)$. This implies:

$$q'_{HL} = y_L + q_{LH} - \frac{1}{p_{LH}^{\gamma}}(1-\Delta), \tag{105}$$

where $y_{LH} \equiv y_L$. Since q_{HH} is different from q_{HL}, then we argue there exists an equilibrium such that $p_{HH} \neq p_{HL} \neq p_{LH}$. In such an equilibrium, we have three types of firms with total quantities q_{HH}, q_{LH}, and q_{HL} with decisions to produce y_H and y_L that are dependent only on the current level of demand.

Using (103) and (105), we can observe:

$$p_{HH} = \left(\frac{1+\Delta}{y_H}\right)^{\frac{1}{\gamma}}, \tag{106}$$

$$p_{HL} = \left(\frac{1+\Delta}{y_L + y_H - p_{LH}^{-\gamma}(1-\Delta)}\right)^{\frac{1}{\gamma}}. \tag{107}$$

Finally one can rewrite the system represented by (95) - (102) as:

$$p_{LH}\eta + \psi_x\left(y_H - p_{LH}^{-\gamma}(1-\Delta)\right) = \psi_y y_L,$$

$$(\psi_y + \beta\psi_x + \beta\psi_y)y_L = \beta\eta\left(p_{LH} + \left(\frac{1+\Delta}{y_L + y_H - p_{LH}^{-\gamma}(1-\Delta)}\right)^{\frac{1}{\gamma}}\right) + \beta\psi_x(y_L + y_H - p_{LH}^{-\gamma}(1-\Delta)),$$

$$(\psi_y + \beta\psi_x)y_H = \beta\psi_x\left(\alpha y_H + (1-\alpha)p_{LH}^{-\gamma}(1-\Delta)\right) + \beta\psi_y(1-\alpha)y_L + \beta\alpha\left(\frac{1+\Delta}{y_H}\right)^{\frac{1}{\gamma}}\eta.$$

In the system above, we need to identify p_{LH}, y_L, and y_H. First, observe:

$$p_{LH} = \left(\frac{(1-\Delta)(\psi_y y_L)^{\gamma}}{(\psi_y y_L)^{\gamma}(y_L + y_H) - (\beta\eta)^{\gamma}(1+\Delta)}\right)^{\frac{1}{\gamma}}, \tag{108}$$

and finally, y_H and y_L are determined by solving:

$$\psi_y y_H = \beta(1-\alpha)y_L(\psi_x + \psi_y) - \beta\psi_x(1-\alpha)\left(\frac{\beta\eta}{\psi_y y_L}\right)^{\gamma}(1+\Delta) + \beta\alpha\eta\left(\frac{1+\Delta}{y_H}\right)^{\frac{1}{\gamma}}, \tag{109}$$

$$(\psi_x + \psi_y)y_L = \left(\frac{(1-\Delta)(\psi_y y_L)^{\gamma}}{(\psi_y y_L)^{\gamma}(y_L + y_H) - (\beta\eta)^{\gamma}(1+\Delta)}\right)^{\frac{1}{\gamma}}\eta + \psi_x\left(\frac{\beta\eta}{\psi_y y_L}\right)^{\gamma}(1+\Delta). \tag{110}$$

B.4 Proof of Result 2

Taylor Expansion Around Steady-State Values. Assume y_H and y_L are at their steady-state values $y_{H,0}$ and $y_{L,0}$ respectively. For small deviations $\Delta\psi_x$ from the steady-state value of ψ_x, we have:

$$y_H \approx y_{H,0} + \frac{\partial y_H}{\partial \psi_x}\Delta\psi_x,$$

$$y_L \approx y_{L,0} + \frac{\partial y_L}{\partial \psi_x}\Delta\psi_x,$$

Substitute into the Expression for p_{LH}. The expression for p_{LH} is:

$$p_{LH} = \left(\frac{(1-\Delta)(\psi_y y_L)^\gamma}{(\psi_y y_L)^\gamma (y_L + y_H) - (\beta\eta)^\gamma (1+\Delta)}\right)^{\frac{1}{\gamma}}.$$

Substitute the Taylor expansions for y_H and y_L:

$$p_{LH} \approx \left(\frac{(1-\Delta)(\psi_y(y_{L,0}+\delta y_L))^\gamma}{(\psi_y(y_{L,0}+\delta y_L))^\gamma\,(y_{L,0}+\delta y_L + y_{H,0} + \delta y_H) - (\beta\eta)^\gamma(1+\Delta)}\right)^{\frac{1}{\gamma}},$$

where:

$$\delta y_H = \frac{\partial y_H}{\partial \psi_x}\Delta\psi_x,$$

$$\delta y_L = \frac{\partial y_L}{\partial \psi_x}\Delta\psi_x.$$

Simplify the Denominator. Expanding the denominator:

$$(\psi_y(y_{L,0}+\delta y_L))^\gamma \approx (\psi_y y_{L,0})^\gamma \left(1 + \gamma\frac{\delta y_L}{y_{L,0}}\right),$$

$$\text{Denominator} \approx (\psi_y y_{L,0})^\gamma\,(y_{L,0} + y_{H,0} + \delta y_L + \delta y_H) - (\beta\eta)^\gamma(1+\Delta),$$

Simplify to:

$$\text{Denominator} \approx (\psi_y y_{L,0})^\gamma (y_{L,0} + y_{H,0}) + (\psi_y y_{L,0})^\gamma(\delta y_L + \delta y_H) - (\beta\eta)^\gamma(1+\Delta).$$

Compute the Derivative. To find $\frac{\partial p_{LH}}{\partial \psi_x}$:

$$\frac{\partial p_{LH}}{\partial \psi_x} \approx p_{LH,0}^{\frac{1}{\gamma}-1} \cdot \frac{\partial}{\partial \psi_x}\left[\frac{(1-\Delta)(\psi_y y_{L,0})^\gamma}{(\psi_y y_{L,0})^\gamma(y_{L,0}+y_{H,0}) + (\psi_y y_{L,0})^\gamma(\delta y_L + \delta y_H) - (\beta\eta)^\gamma(1+\Delta)}\right],$$

$$\frac{\partial}{\partial \psi_x}\left[\frac{A}{B}\right] = \frac{B\frac{\partial A}{\partial \psi_x} - A\frac{\partial B}{\partial \psi_x}}{B^2},$$

where $A = (1-\Delta)(\psi_y y_{L,0})^\gamma$ and $B = (\psi_y y_{L,0})^\gamma(y_{L,0}+y_{H,0}) + (\psi_y y_{L,0})^\gamma(\delta y_L + \delta y_H) - (\beta\eta)^\gamma(1+\Delta)$.

Since $\frac{\partial A}{\partial \psi_x} = 0$ and $\frac{\partial B}{\partial \psi_x} = (\psi_y y_{L,0})^\gamma \left(\frac{\partial y_L}{\partial \psi_x} + \frac{\partial y_H}{\partial \psi_x}\right)$:

$$\frac{\partial p_{LH}}{\partial \psi_x} \approx -p_{LH,0}^{\frac{1}{\gamma}-1} \cdot \frac{(\psi_y y_{L,0})^\gamma \left(\frac{\partial y_L}{\partial \psi_x} + \frac{\partial y_H}{\partial \psi_x}\right)}{\left[(\psi_y y_{L,0})^\gamma (y_{L,0} + y_{H,0}) - (\beta\eta)^\gamma (1+\Delta)\right]^2}.$$

Condition for Decreasing p_{LH}. For $\frac{\partial p_{LH}}{\partial \psi_x} < 0$:

$$(\psi_y y_{L,0})^\gamma \left(\frac{\partial y_L}{\partial \psi_x} + \frac{\partial y_H}{\partial \psi_x}\right) > 0.$$

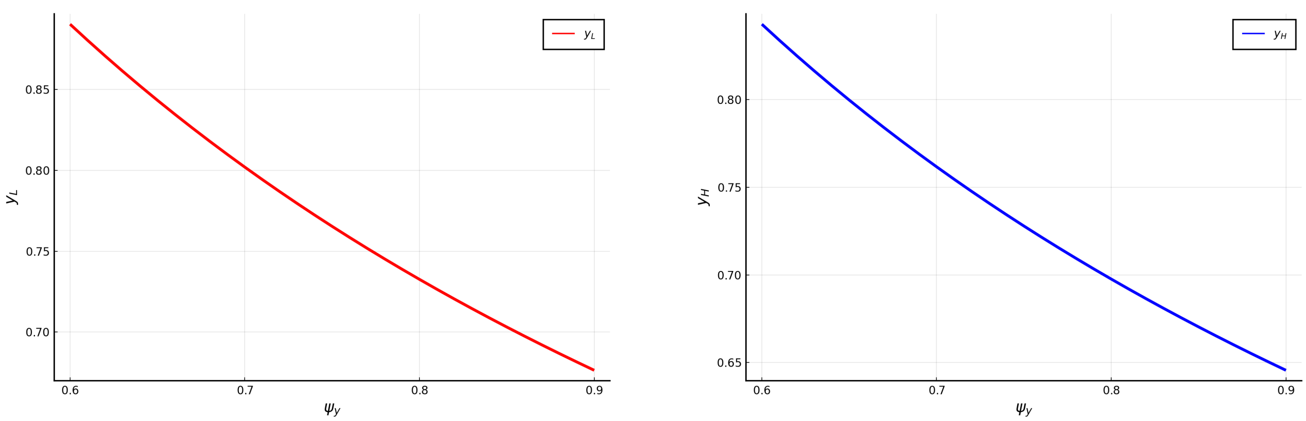
This implies:

$$\frac{\partial y_L}{\partial \psi_x} + \frac{\partial y_H}{\partial \psi_x} > 0.$$

Therefore, p_{LH} decreases with ψ_x if the sum of the derivatives of y_L and y_H with respect to ψ_x is positive.

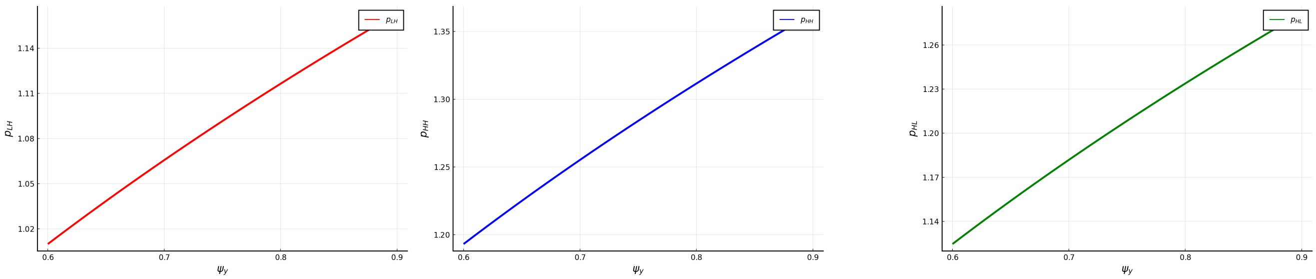
C Equilibrium conditions for different values of ψ_y

We now turn to the results for the equilibrium conditions for different values ψ_y:



(a) Production in the Low-Demand state (L). (b) Production in the High-Demand state (H).

Figure 20: Production for different values of ψ_y.



(a) The price in L, considering the previous state was H.

(b) The price in H, considering the previous state was H.

(c) The price in H, considering the previous state was L.

Figure 21: Prices for different values of ψ_y.

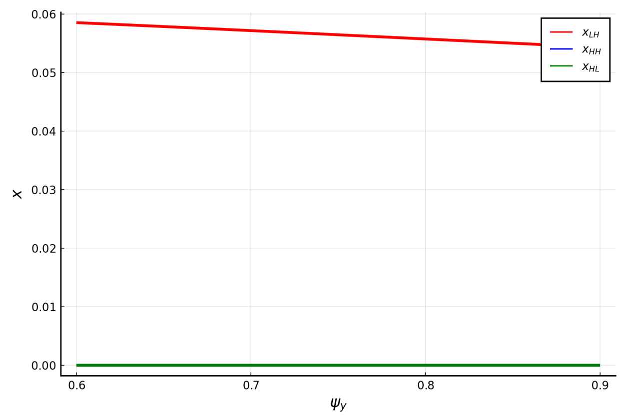
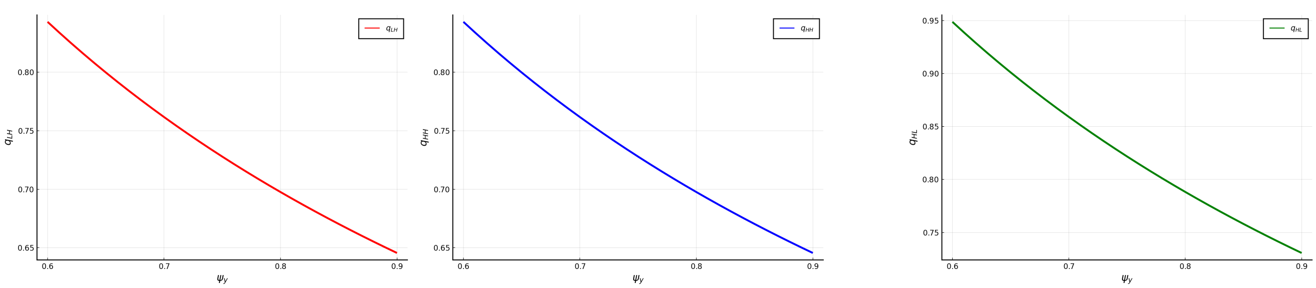


Figure 22: Inventories across various states and different values of ψ_y.



(a) The quantity in L, considering the previous state was H.

(b) The quantity in H, considering the previous state was H.

(c) The quantity in H, considering the previous state was L.

Figure 23: Quantities for different values of ψ_y.

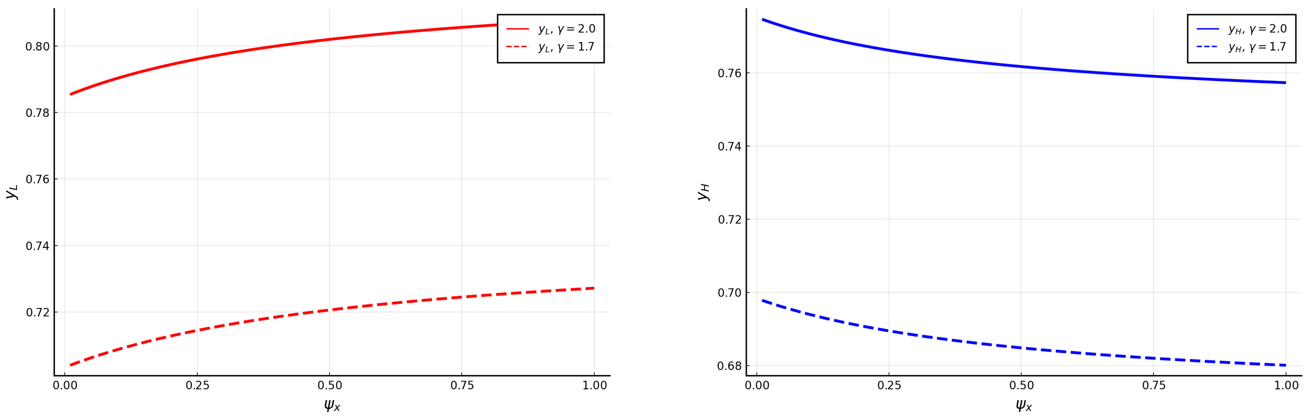
D Equilibrium conditions for different values of γ

Consider now the following parameters in Table 6:

Parameter	Description	Value
ψ_x	Cost to carry inventories	0.5
ψ_y	Cost to produce	0.7
α	Probability to continue in High-Demand	0.3
β	Discount factor	0.95
Δ	Demand shock	0.2
η	Market power	0.5

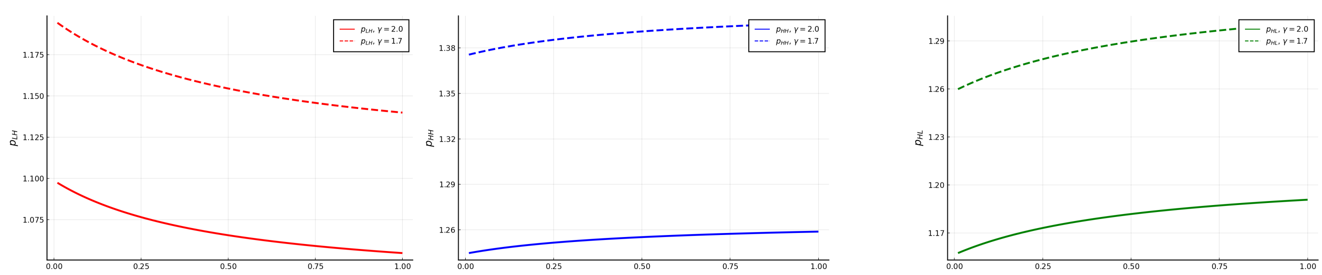
Table 6: Model Parameters.

We now turn to the results for the equilibrium conditions for different values ψ_x and for two different values of elasticity of demand γ:



(a) Production in the Low-Demand state (L). (b) Production in the High-Demand state (H).

Figure 24: Production for different values of ψ_x and two different values of γ.



(a) The price in L, considering the previous state was H. (b) The price in H, considering the previous state was H. (c) The price in H, considering the previous state was L.

Figure 25: Prices for different values of ψ_x and two different values of γ.

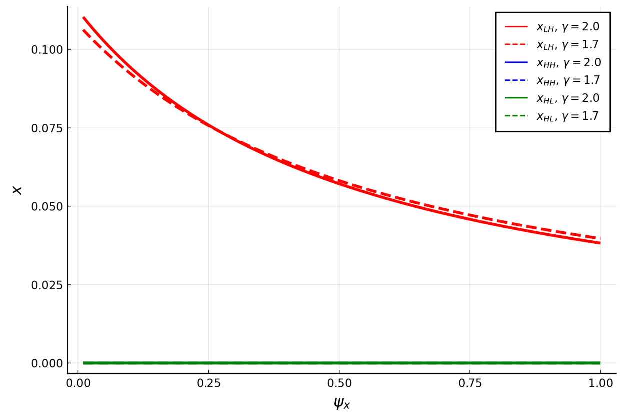


Figure 26: Inventories across various states and different values of ψ_x and two different values of γ.

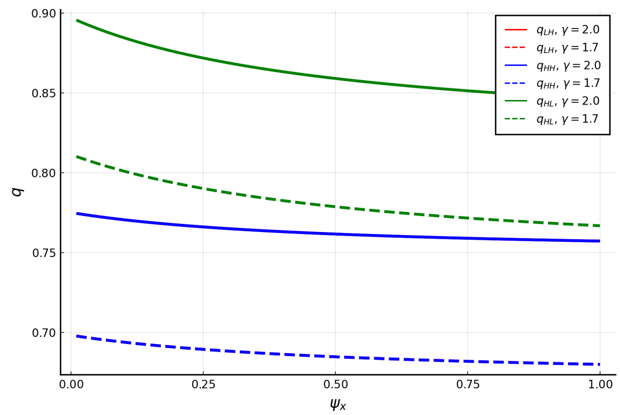
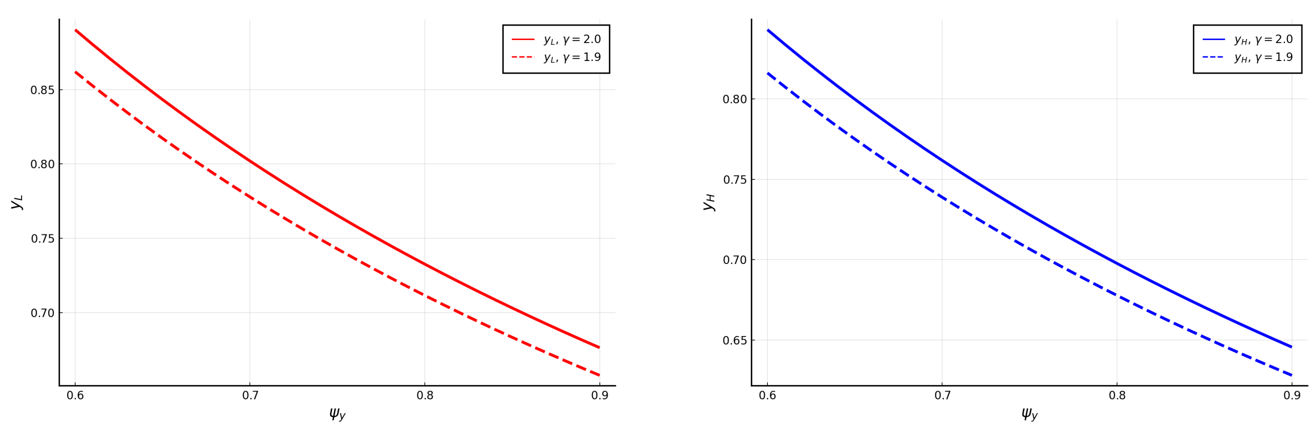


Figure 27: Quantities for different values of ψ_x and two different values of γ.

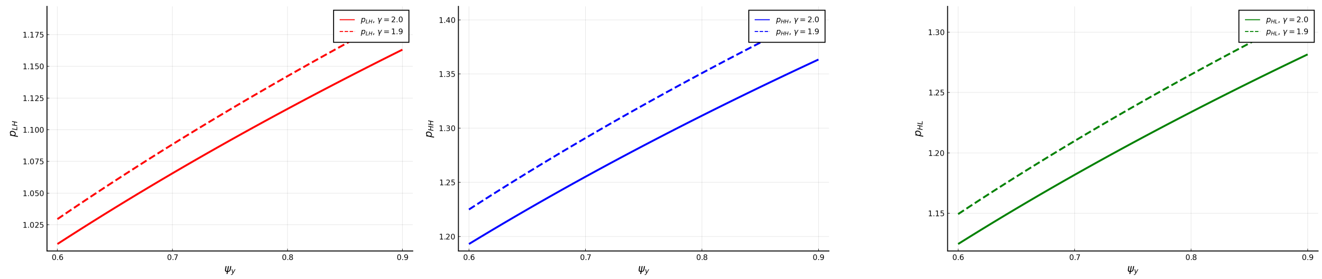
Below we show the results for the equilibrium conditions for different values ψ_y and for two different values of elasticity of demand γ:



(a) Production in the Low-Demand state (L).

(b) Production in the High-Demand state (H).

Figure 28: Production for different values of ψ_y and two different values of γ.



(a) The price in L, considering the previous state was H.

(b) The price in H, considering the previous state was H.

(c) The price in H, considering the previous state was L.

Figure 29: Prices for different values of ψ_y and two different values of γ.

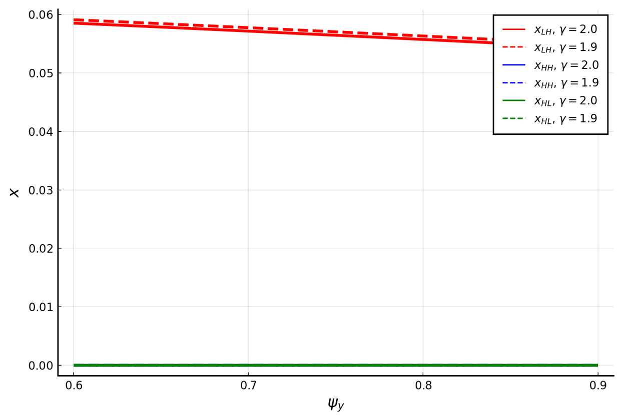


Figure 30: Inventories across various states and different values of ψ_y and two different values of γ.

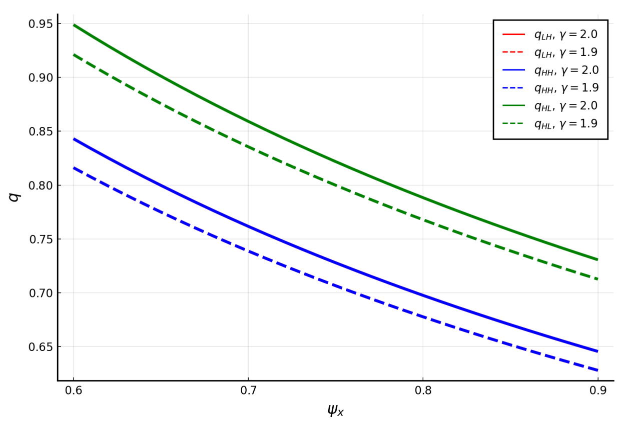


Figure 31: Quantities for different values of ψ_y and two different values of γ.

E Dynamics in the Simple Model

E.1 Dynamics for Different States in the Simple Model

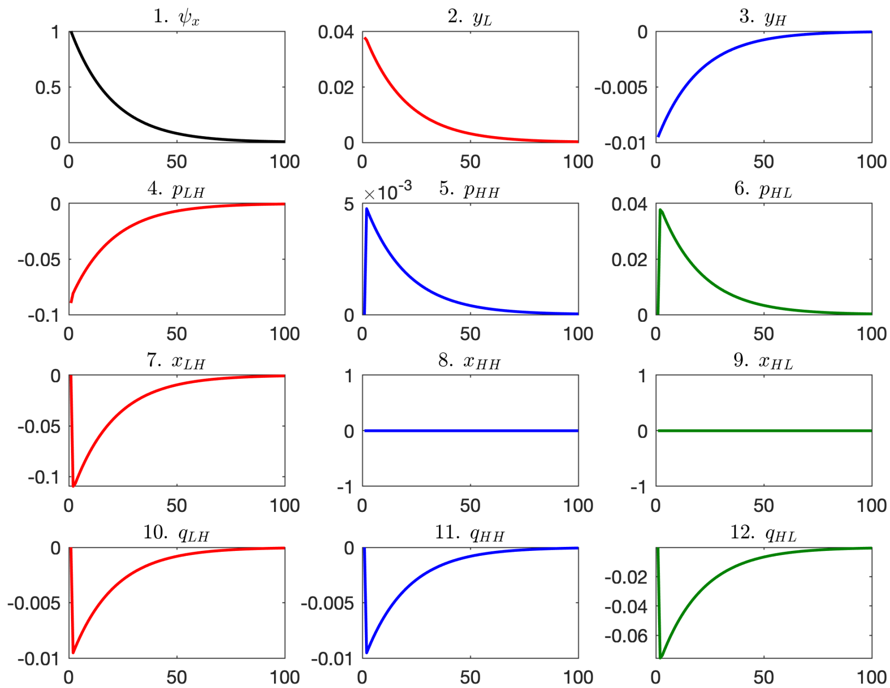


Figure 32: Impulse Response Functions for the selected variables following a positive shock to ψ_x.

E.2 Dynamics after a negative shock in β

Below we show the same results for a persistent fall in β for 100 periods, after a fall of 30 % on impact.

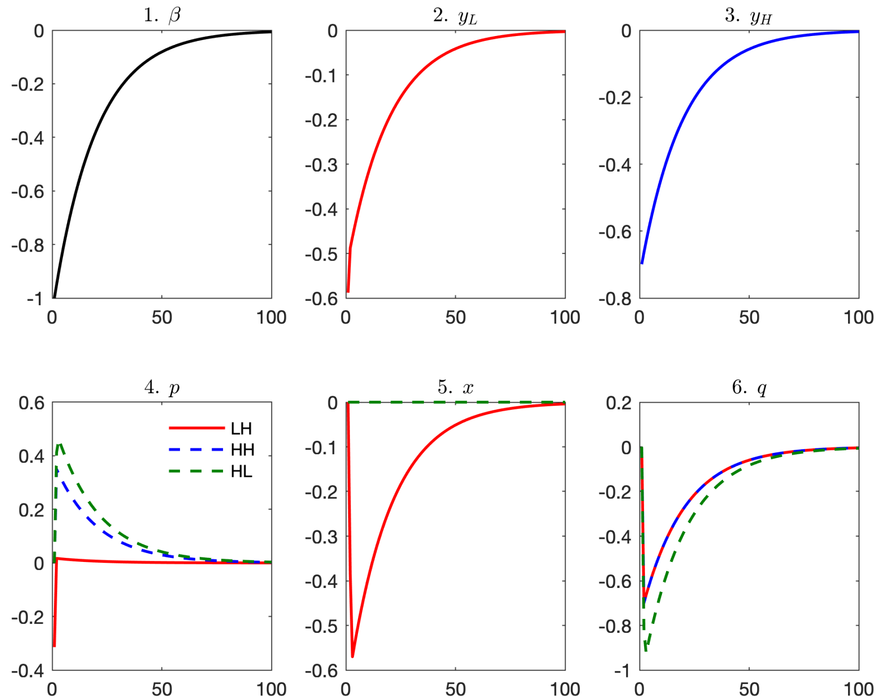


Figure 33: Comparison of Impulse Response Functions for selected variables following a negative shock in β.

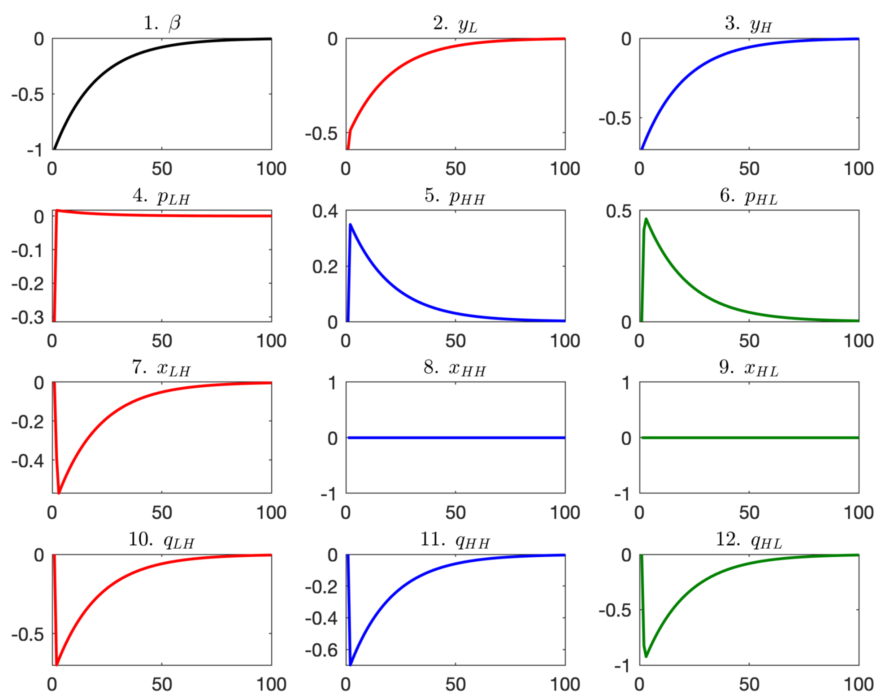


Figure 34: Impulse Response Functions for the selected variables following a negative shock to β.

E.3 Second-order moments in the Simple Economy

Table 7 summarizes the fist and second moments of the main variables in each of the cases analyzed in Section 3.3. For each variable we report the steady state value, reported as "Mean" and the standard deviation, referred here as "Std".

		ψ_x	β
y_L	Mean	0.813	0.813
	Std	0.001	0.016
y_H	Mean	0.846	0.846
	Std	0.000	0.022
p_{LH}	Mean	0.904	0.904
	Std	0.003	0.003
p_{HH}	Mean	1.332	1.332
	Std	0.000	0.011
p_{HL}	Mean	1.197	1.197
	Std	0.001	0.015
x_{LH}	Mean	0.234	0.234
	Std	0.004	0.018
q_{LH}	Mean	0.846	0.846
	Std	0.000	0.022
q_{HH}	Mean	0.846	0.846
	Std	0.000	0.022
q_{HL}	Mean	1.046	1.046
	Std	0.002	0.030
		Correlations	
corr(p_{LH},y_L)		-0.9997	0.1905
corr(p_{HH},y_H)		-0.9493	-0.9493
corr(p_{HL},y_H)		-0.9492	-0.9479

Table 7: First and second moments for key variables under the following scenarios: a positive shock in ψ_x and a negative shock in β.

We can observe that Table 7 confirms the IRFs regarding the selected variables. The volatility of the variables is higher for the shock in β.

F Inventories in the demand function

Now consider the following demand function:

$$S_t(p_t, x_t) = \bar{S}(p_t)x_t^{\xi}, \tag{111}$$

where the only difference between this specification and the one represented by (7) is that in this case holding inventories boost total demand. The timeline and the specification of the model will be the same as above.

Therefore, the firm's problem at period t can be stated as:

$$V(q_t) = \max_{\{p_t, y_t, x_t\}} p_t S_t(p_t, x_t) - \psi_y \frac{y_t^2}{2} - \psi_x \frac{x_{t+1}^2}{2} + \beta \mathbb{E}\left[V(q_{t+1})\right], \tag{112}$$

subject to:

$$q_{t+1} = q_t - S_t(p_t, x_t) + y_t, \tag{113}$$

$$x_{t+1} \geq 0. \tag{114}$$

In recursive formulation the problem above can be written as:

$$V(q) = \max_{\{p,y\}} pS(p, q - y_{-1}) - \psi_y \frac{y^2}{2} - \psi_x \frac{(q - S(p, q - y_{-1}))^2}{2} + \beta \mathbb{E}\left[V(q')\right], \tag{115}$$

subject to:

$$q' = q - S(p, q - y_{-1}) + y, \tag{116}$$

$$x' \geq 0. \tag{117}$$

Denote by μ the Lagrange multiplier associated with the inventories constraint given by Equation (117).

By taking first-order conditions with respect to p and y, we obtain the following results, respectively:

$$\frac{S(p, q - y_{-1}) + pS_1(p, q - y_{-1})}{S_1(p, q - y_{-1})} + \psi_x(q - S(p, q - y_{-1})) = \beta \mathbb{E}[V'(q')] + \mu,$$

$$\psi_y y = \beta \mathbb{E}[V'(q')].$$

Notice that $V'(q) = pS_2(p, q - y_{-1}) + (1 - S_2(p, q - y_{-1}))\left(-\psi_x(q - S(p, q - y_{-1})) + \beta \mathbb{E}[V'(q')] + \mu\right)$. Therefore, using the envelope condition and the aforementioned first-order conditions, we can summarize the solution to this problem by the following equations:

$$\frac{S(p, q - y_{-1}) + pS_1(p, q - y_{-1})}{S_1(p, q - y_{-1})} + \psi_x(q - S(p, q - y_{-1})) = \psi_y y + \mu, \tag{118}$$

$$\psi_y y = \beta \mathbb{E}\left[p' S_2(p', q' - y) + (1 - S_2(p', q' - y))\left(-\psi_x(q' - S(p', q' - y)) + \psi_y y' + \mu'\right)\right], \tag{119}$$

$$q' = q - S(p, q - y_{-1}) + y, \tag{120}$$

$$\mu(q - S(p, q - y_{-1})) = 0. \tag{121}$$

Observe the elasticity price of demand is given by $E_p^S = \dfrac{pS_1(p, q - y_{-1})}{S(p, q - y_{-1})}$, which implies we can rewrite $\dfrac{S(p) + pS_1(p, q - y_{-1})}{S_1(p, q - y_{-1})} = \dfrac{S(p, q - y_{-1})}{S_1(p, q - y_{-1})}(1 + E_p^S)$.

Using this expression and manipulating equations (118) - (121) we have:

$$\psi_y y = \beta \mathbb{E}[p' S_2(p', q' - y)] + \beta \frac{S(p, q - y_{-1})}{S_1(p, q - y_{-1})}(1 + E_p^S)\mathbb{E}[(1 - S_2(p', q' - y))] - \beta \psi_x y \mathbb{E}[(1 - S_2(p', q' - y))] + \beta \psi_x \mathbb{E}[(1 - S_2(p', q' - y))S(p', q' - y)] + \beta \psi_y \mathbb{E}[(1 - S_2(p', q' - y))(y' - y)] + \beta \mathbb{E}[(1 - S_2(p', q' - y))(\mu' - \mu)]. \tag{122}$$

Observe that in equilibrium we need to have $x > 0$, which implies $\mu = 0$, otherwise we would have $pS(p, q - y_{-1}) = 0$. Assuming $\bar{S}(p) = \frac{1}{p^\gamma}$, and observing: $q_{t+1} = y_t + q_t - \frac{x_t^\xi}{p_t^\gamma}$. Therefore in equilibrium we have:

$$y = \frac{x^\xi}{p^\gamma}.$$

Also observe we have $q = x + \frac{x^\xi}{p^\gamma}$. Using these observations, the equilibrium conditions of the model are:

$$\psi_y y = \beta \frac{p \xi (q - y)^{\xi - 1}}{p^\gamma} + \beta \left(\frac{p^\gamma - \xi (q - y)^{\xi - 1}}{p^\gamma} \right) p \left(\frac{\gamma - 1}{\gamma} \right), \tag{123}$$

$$y = \frac{(q - y)^\xi}{p^\gamma}, \tag{124}$$

$$p \left(\frac{\gamma - 1}{\gamma} \right) + \psi_x \left(q - \frac{(q - y)^\xi}{p^\gamma} \right) = \psi_y y. \tag{125}$$

Take a simple case where we assume $\xi = \gamma = 1$. In this case the equilibrium, conditions are the following ones:

$$p = \frac{\psi_y}{\psi_x},$$
$$y = \frac{\beta}{\psi_y},$$
$$q = \frac{\beta(\psi_y + \psi_x)}{\psi_y \psi_x},$$
$$x = \frac{\beta}{\psi_x}.$$

The signal of the derivatives with respect to ψ_x are such that:

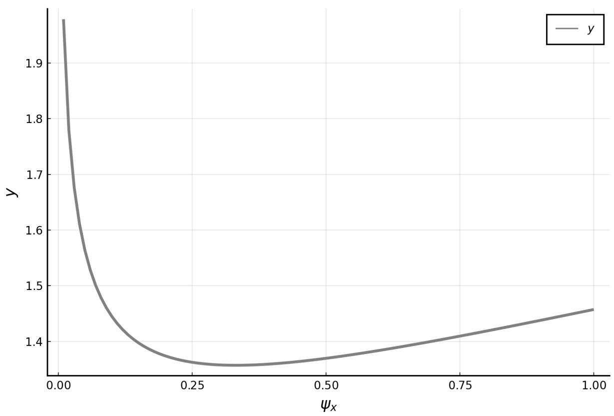
$$\frac{\partial p}{\partial \psi_x} < 0, \quad \frac{\partial y}{\partial \psi_x} = 0, \quad \frac{\partial q}{\partial \psi_x} < 0, \quad \frac{\partial x}{\partial \psi_x} < 0.$$

Take the following parameters in Table 8:

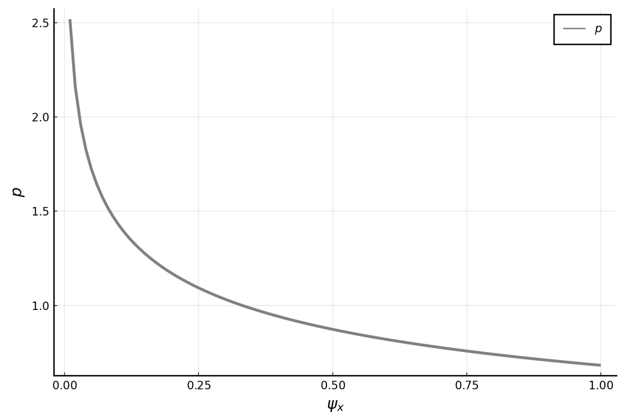
The results below show the equilibrium conditions for different values ψ_x:

Parameter	Description	Value
ψ_x	Cost to carry inventories	0.5
ψ_y	Cost to produce	0.7
ξ	Inventories in the demand	1.0
β	Discount factor	0.95
γ	Elasticity of demand with respect to price	2.0

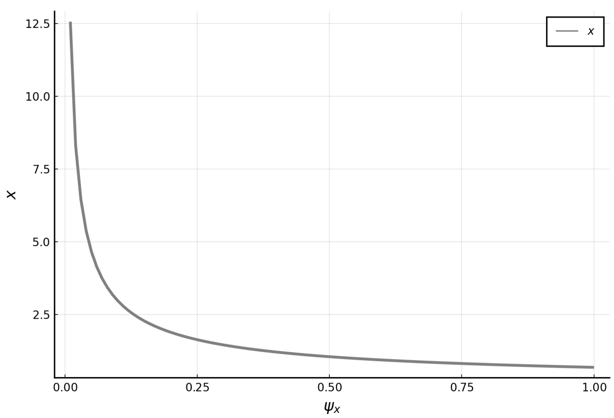
Table 8: Model Parameters.



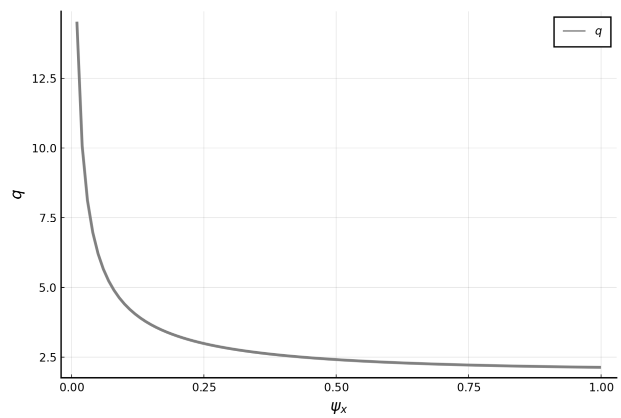
(a) Production.



(b) Price.



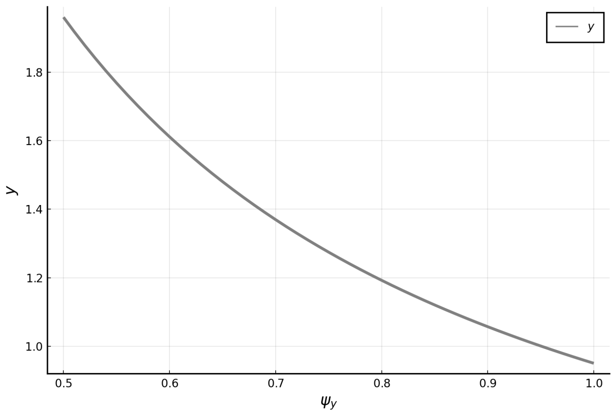
(c) Inventories.



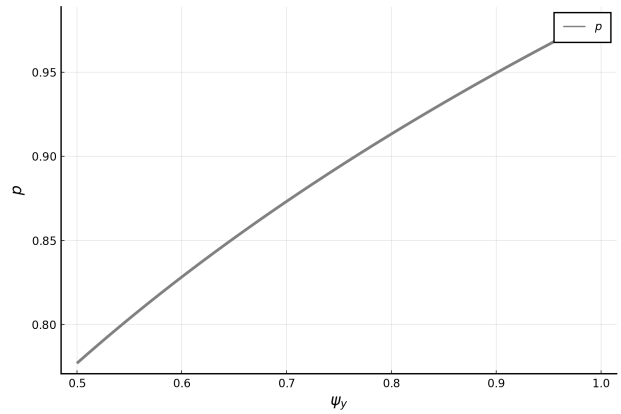
(d) Quantity.

Figure 35: Equilibrium conditions for different values of ψ_x.

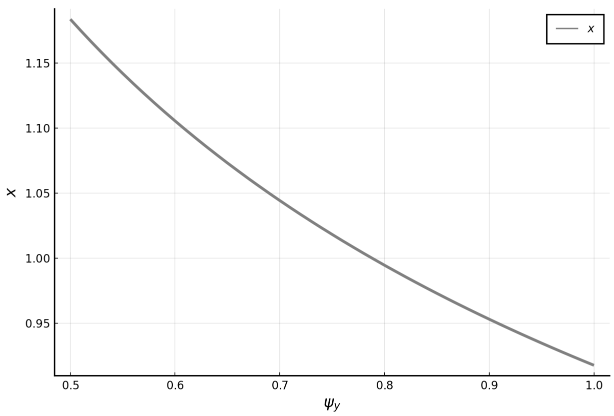
Now we show the results of the equilibrium conditions for different values ψ_y:



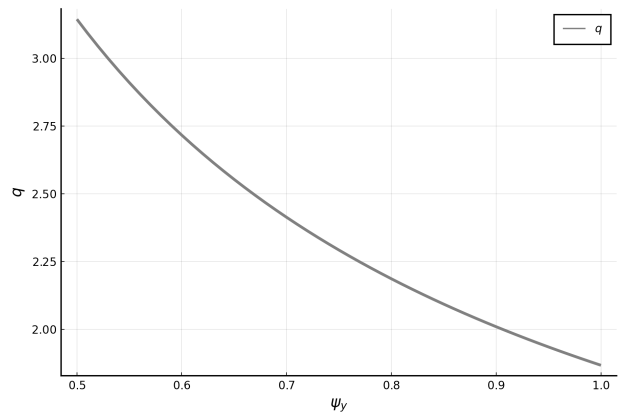
(a) Production.



(b) Price.



(c) Inventories.



(d) Quantity.

Figure 36: Equilibrium conditions for different values of ψ_y.

We now present the dynamics of the system after a Monetary Policy shock affecting either ψ_x or β considering the calibration in Table 8. To ease the comparison and be able to see the differences between the business cycle in both situations we keep the shock to be the same for both cases. Figure 37 shows the Impulse Response Functions for a persistent increase in ψ_x for the production, prices, quantities, and inventories, respectively.

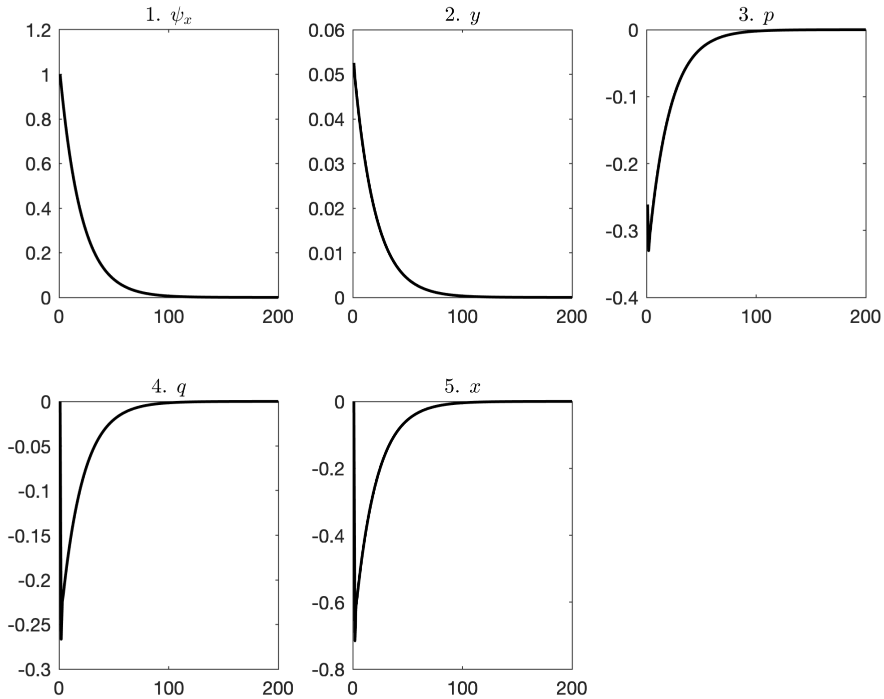


Figure 37: Impulse Response Functions for the selected variables following a positive shock to ψ_x.

Notice that each panel reports the proportional change for the variable under consideration, in percentage points. For instance, Panel 1 reports a persistent increase in ψ_x for 100 periods, after an increase of 1 % on impact, except for the inventories, for which we compute the absolute variation.

Below we show the same results for a persistent fall in β for 100 periods, after a fall of 1 % on impact.

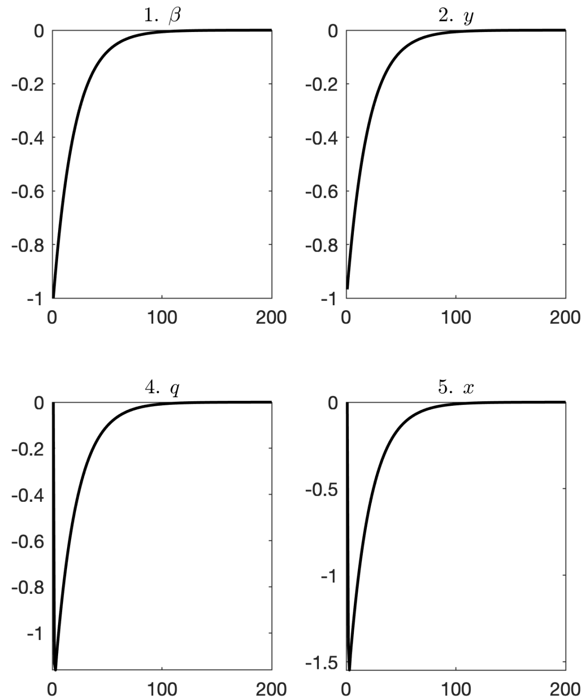


Figure 38: Impulse Response Function for the selected variables after a negative shock in β.

G Inventories in the demand function with demand shocks

Now consider the following demand function:

$$S_t(p_t, x_t) = \bar{S}(p_t) x_t^{\xi} \varepsilon_t, \tag{126}$$

where the structure of the shocks and the transition matrix will have the same structure as the one we considered above: a High-Demand shock (H) and a Low-Demand shock (L), denoted as:

$$\varepsilon = \begin{cases} 1 + \Delta & \text{if shock } = H, \\ 1 - \Delta & \text{if shock } = L. \end{cases}$$

In the scenario of a High-Demand shock (H), there's a probability of α for the economy to remain in the H state, with the remaining probability $(1 - \alpha)$ indicating a transition to the Low-Demand state (L). Conversely, when the economy is in the L state, the probability of transitioning to the H state is 1, while the probability of staying in the L state is 0. We summarize these transition probabilities in the following transition matrix:

$$\Pi = \begin{bmatrix} \pi(H|H) & \pi(L|H) \\ \pi(H|L) & \pi(L|L) \end{bmatrix} = \begin{bmatrix} \alpha & 1 - \alpha \\ 1 & 0 \end{bmatrix}.$$

Using the notation $x_{IJ} = x(I|J)$, where x_{IJ} denotes the realization of variable x in state I given the state J, and applying (119) for the case when the economy was in state H and is currently in state L, leads to:

$$\psi_y y_{LH} = \beta[p_{HL} S_2(p_{HL}, q_{HL} - y_{LH}) + (1 - S_2(p_{HL}, q_{HL} - y_{LH}))(-\psi_x(q_{HL} - S(p_{HL}, q_{HL} - y_{LH})) + \psi_y y_{HL} + \mu_{HL})], \tag{127}$$

where by observing $q_{HL} = q_{LH} - S(p_{LH}, q_{LH} - y_H) + y_{LH}$ and using (118) we get:

$$\frac{S(p_{LH}, q_{LH} - y_H)}{S_1(p_{LH}, q_{LH} - y_H)}(1 + E^S_{p_{LH}}) + \psi_x(q_{LH} - S(p_{LH}, q_{LH} - y_H)) = \psi_y y_{LH} + \mu_{LH},$$

where by replacing this result in (127):

$$\psi_y y_{LH} = \beta\Bigg[p_{HL} S_2(p_{HL}, q_{HL} - y_{LH}) + (1 - S_2(p_{HL}, q_{HL} - y_{LH}))\bigg(\frac{S(p_{LH}, q_{LH} - y_H)}{S_1(p_{LH}, q_{LH} - y_H)}(1 + E^S_{p_{LH}}) - \psi_y y_{LH} - \psi_x y_{LH} + \psi_x S(p_{HL}, q_{HL} - y_{LH}) + \psi_y y_{HL} + \mu_{HL}\bigg)\Bigg]. \tag{128}$$

Now do the same for the case the economy was in state L and is currently in state H:

$$\begin{aligned}
\psi_y y_{HL} = {} & \beta[\alpha p_{HH} S_2(p_{HH}, q_{HH} - y_{HL}) + (1-\alpha) p_{LH} S_2(p_{LH}, q_{LH} - y_{HL})] \\
& - \beta\psi_x(q_{HL} - S(p_{HL}, q_{HL} - y_{LH}))[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HL})) + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HL}))] \\
& - \beta\psi_x y_{HL}[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HL})) + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HL}))] \\
& + \beta\psi_x[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HL}))S(p_{HH}, q_{HH} - y_{HL}) + \\
& \qquad (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HL}))S(p_{LH}, q_{LH} - y_{HL})] \\
& + \beta\psi_y[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HL}))y_{HH} + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HL}))y_{LH}] \\
& + \beta[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HL}))\mu_{HH} + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HL}))\mu_{LH}].
\end{aligned} \tag{129}$$

Using the same idea for the case the economy was in state H and is currently in state H we reach:

$$\begin{aligned}
\psi_y y_{HH} = {} & \beta[\alpha p_{HH} S_2(p_{HH}, q_{HH} - y_{HH}) + (1-\alpha) p_{LH} S_2(p_{LH}, q_{LH} - y_{HH})] \\
& - \beta\psi_x(q_{HH} - S(p_{HH}, q_{HH} - y_H))[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HH})) + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HH}))] \\
& - \beta\psi_x y_{HH}[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HH})) + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HH}))] \\
& + \beta\psi_x[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HH}))S(p_{HH}, q_{HH} - y_{HH}) + \\
& \qquad (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HH}))S(p_{LH}, q_{LH} - y_{HH})] \\
& + \beta\psi_y[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HH}))y_{HH} + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HH}))y_{LH}] \\
& + \beta[\alpha(1 - S_2(p_{HH}, q_{HH} - y_{HH}))\mu_{HH} + (1-\alpha)(1 - S_2(p_{LH}, q_{LH} - y_{HH}))\mu_{LH}].
\end{aligned} \tag{130}$$

Using (118) we get:

$$\frac{S(p_{HH}, q_{HH} - y_H)}{S_1(p_{HH}, q_{HH} - y_H)}(1 + E^S_{p_{HH}}) + \psi_x(q_{HH} - S(p_{HH}, q_{HH} - y_H)) = \psi_y y_{HH}, \tag{131}$$

$$\frac{S(p_{HL}, q_{HL} - y_{LH})}{S_1(p_{HL}, q_{HL} - y_{LH})}(1 + E^S_{p_{HL}}) + \psi_x(q_{HL} - S(p_{HL}, q_{HL} - y_{LH})) = \psi_y y_{HL}, \tag{132}$$

$$\frac{S(p_{LH}, q_{LH} - y_H)}{S_1(p_{LH}, q_{LH} - y_H)}(1 + E^S_{p_{LH}}) + \psi_x(q_{LH} - S(p_{LH}, q_{LH} - y_H)) = \psi_y y_{LH}. \tag{133}$$

Now observe we have the following:

$$q'_{HH} = q_{HH} - S(p_{HH}, q_{HH} - y_{HH}) + y_{HH},$$

$$q'_{HH} = q_{HL} - S(p_{HL}, q_{HL} - y_{LH}) + y_{HL},$$
$$q'_{HL} = q_{LH} - S(p_{LH}, q_{LH} - y_{HH}) + y_{LH},$$
$$q'_{HL} = q_{LH} - S(p_{LH}, q_{LH} - y_{HL}) + y_{LH},$$

which means $y_{HH} = y_{HL} \equiv y_H$. Lastly notice:

$$q'_{LH} = q_{HH} - S(p_{HH}, q_{HH} - y_{HH}) + y_{HH},$$

$$q'_{LH} = q_{HL} - S(p_{HL}, q_{HL} - y_{LH}) + y_{HL},$$

which means $q'_{HH} = q'_{LH}$. Using the observations above, (131) and (132) it is easy to show that $p_{HH} = p_{HL} = p_H$. Now using the following notations: $y_{LH} \equiv y_L$, $p_{LH} = p_L$, $q'_{HH} = q'_{LH} \equiv q_{-H}$, and $q'_{HL} \equiv q_{-L}$ and that in equilibrium $y_H = S(p_H, q_{-H} - y_H)$, the equilibrium conditions from the system represented by equations (128) - (133) is given by:

$$\begin{aligned}\psi_y y_L = \beta\Bigg[& p_H S_2(p_H, q_{-L} - y_L) + (1 - S_2(p_H, q_{-L} - y_L))\Bigg(\frac{S(p_L, q_{-H} - y_H)}{S_1(p_L, q_{-H} - y_H)}(1 + E^S_{p_L}) - \psi_y y_L \\ & - \psi_x y_L + \psi_x S(p_H, q_{-L} - y_L) + \psi_y y_H\Bigg)\Bigg].\end{aligned} \tag{134}$$

$$\begin{aligned}\psi_y y_H = {} & \beta[\alpha p_H S_2(p_H, q_{-H} - y_H) + (1-\alpha) p_L S_2(p_L, q_{-H} - y_H)] \\ & - \beta\psi_x(q_{-L} - S(p_H, q_{-L} - y_L))[\alpha(1 - S_2(p_H, q_{-H} - y_H)) + (1-\alpha)(1 - S_2(p_L, q_{-H} - y_H))] \\ & - \beta\psi_x y_H[\alpha(1 - S_2(p_H, q_{-H} - y_H)) + (1-\alpha)(1 - S_2(p_L, q_{-H} - y_H))] \\ & + \beta\psi_x[\alpha(1 - S_2(p_H, q_{-H} - y_H))S(p_H, q_{-H} - y_H) + \\ & \qquad (1-\alpha)(1 - S_2(p_L, q_{-H} - y_H))S(p_L, q_{-H} - y_H)] \\ & + \beta\psi_y[\alpha(1 - S_2(p_H, q_{-H} - y_H))y_H + (1-\alpha)(1 - S_2(p_L, q_{-H} - y_H))y_L].\end{aligned} \tag{135}$$

$$\frac{S(p_H, q_{-H} - y_H)}{S_1(p_H, q_{-H} - y_H)}(1 + E^S_{p_H}) + \psi_x(q_{-H} - S(p_H, q_{-H} - y_H)) = \psi_y y_H. \tag{136}$$

$$\frac{S(p_L, q_{-H} - y_H)}{S_1(p_L, q_{-H} - y_H)}(1 + E^S_{p_L}) + \psi_x(q_{-H} - S(p_L, q_{-H} - y_H)) = \psi_y y_L. \tag{137}$$

$$y_H = S(p_H, q_{-H} - y_H). \tag{138}$$

$$q_{-L} = q_{-H} - S(p_L, q_{-H} - y_H) + y_L. \tag{139}$$

By taking the case where $\Delta = 0$ and $\xi = \gamma = 1$ we reach the same solution we had in the simple case represented by the solution to the problem (123) - (125):

$$\begin{aligned} p_H = p_L &= \frac{\psi_y}{\psi_x}, \\ y_H = y_L &= \frac{\beta}{\psi_y}, \\ q_{-H} = q_{-L} &= \frac{\beta(\psi_y + \psi_x)}{\psi_y \psi_x}, \\ x_H = x_L &= \frac{\beta}{\psi_x}. \end{aligned}$$

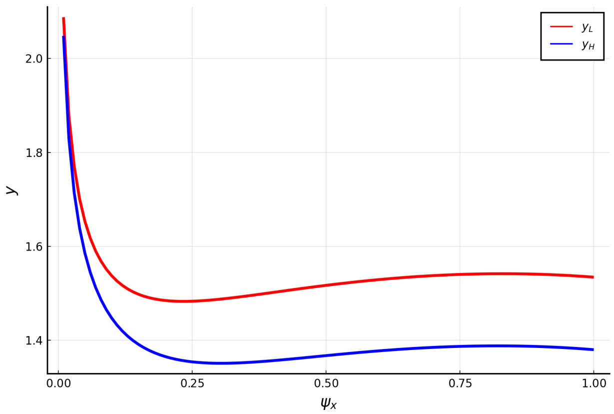
By letting $\xi = \gamma = 1$ we are able to solve the system represented by (134) - (139) and we can show that $\frac{\partial p_i}{\partial \psi_x} < 0$ and $\frac{\partial x_i}{\partial \psi_x} < 0$, for $i \in \{L, H\}$.

Take the following parameters in Table 9:

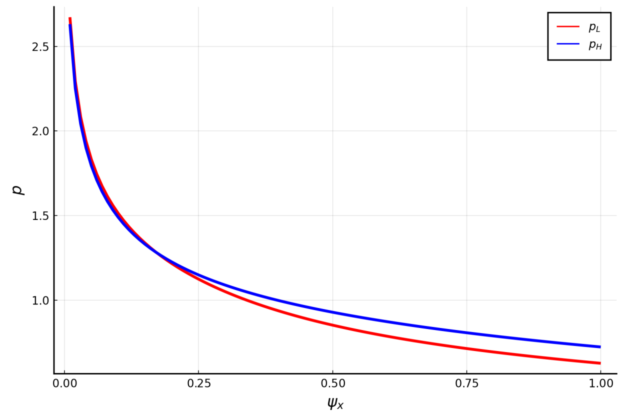
Parameter	**Description**	**Value**
ψ_x	Cost to carry inventories	0.5
ψ_y	Cost to produce	0.7
α	Probability to continue in High-Demand	0.5
ξ	Inventories in the demand	1.0
β	Discount factor	0.95
Δ	Demand shock	0.2
γ	Elasticity of demand with respect to price	2.0

Table 9: Model Parameters.

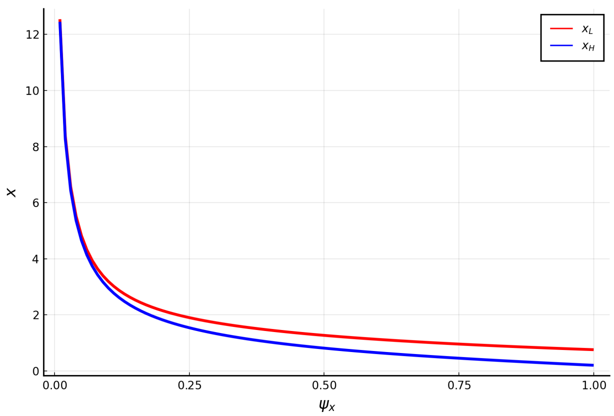
The results below show the equilibrium conditions for different values ψ_x:



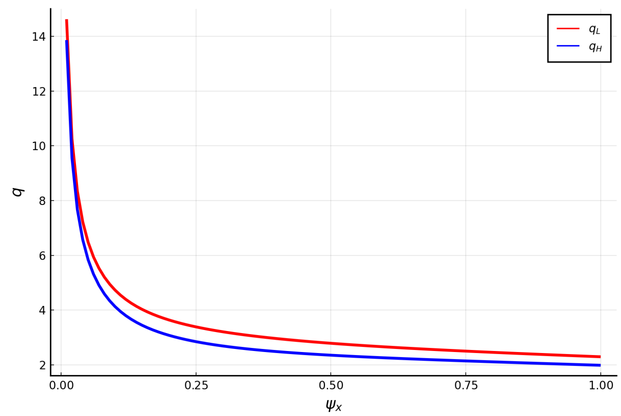
(a) Production for different values of ψ_x.



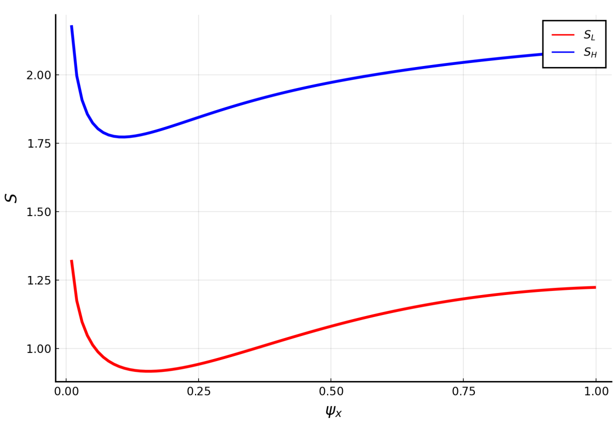
(b) Price for different values of ψ_x.



(c) Inventories for different values of ψ_x.



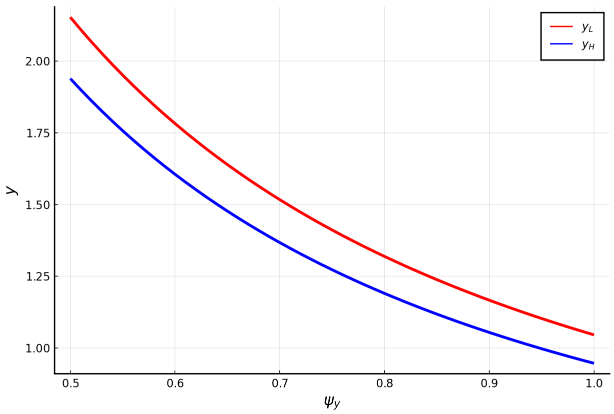
(d) Quantity for different values of ψ_x.



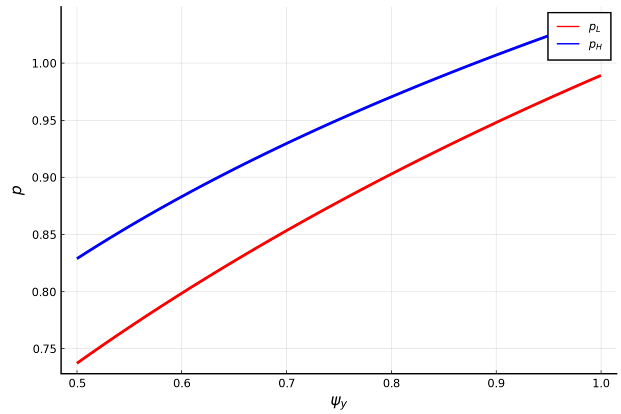
(e) Demand for different values of ψ_x.

Figure 39: Equilibrium conditions for different values of ψ_x across different states.

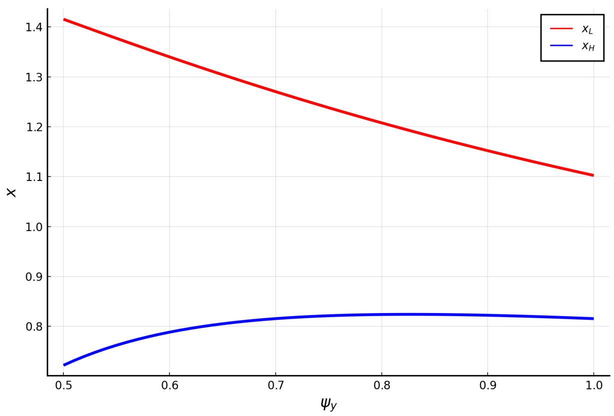
Now we show the results of the equilibrium conditions for different values ψ_y:



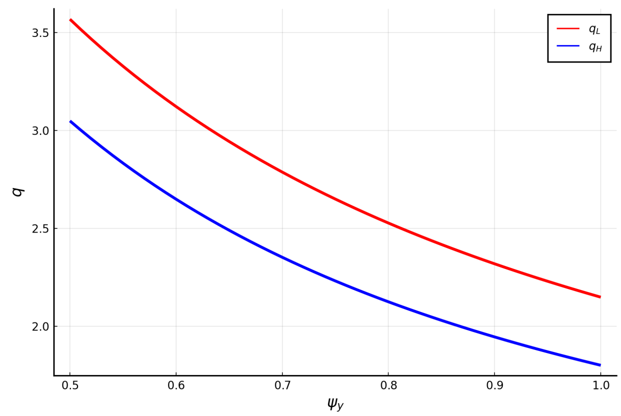
(a) Production for different values of ψ_y.



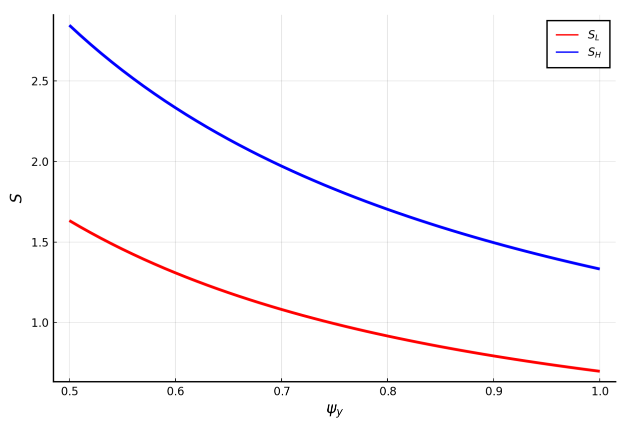
(b) Price for different values of ψ_y.



(c) Inventories for different values of ψ_y.



(d) Quantity for different values of ψ_y.



(e) Demand for different values of ψ_y.

Figure 40: Equilibrium conditions for different values of ψ_y across different states.

We now present the dynamics of the system after a Monetary Policy shock affecting either ψ_x or β considering the calibration in Table 9. To ease the comparison and be able to see the differences between the business cycle in both situations we keep the shock to be the same for both cases. Figure 41 shows the Impulse Response Functions for a persistent increase in ψ_x for the production, prices, quantities, and inventories, respectively. Figure 42 shows the same results but comparing the different states.

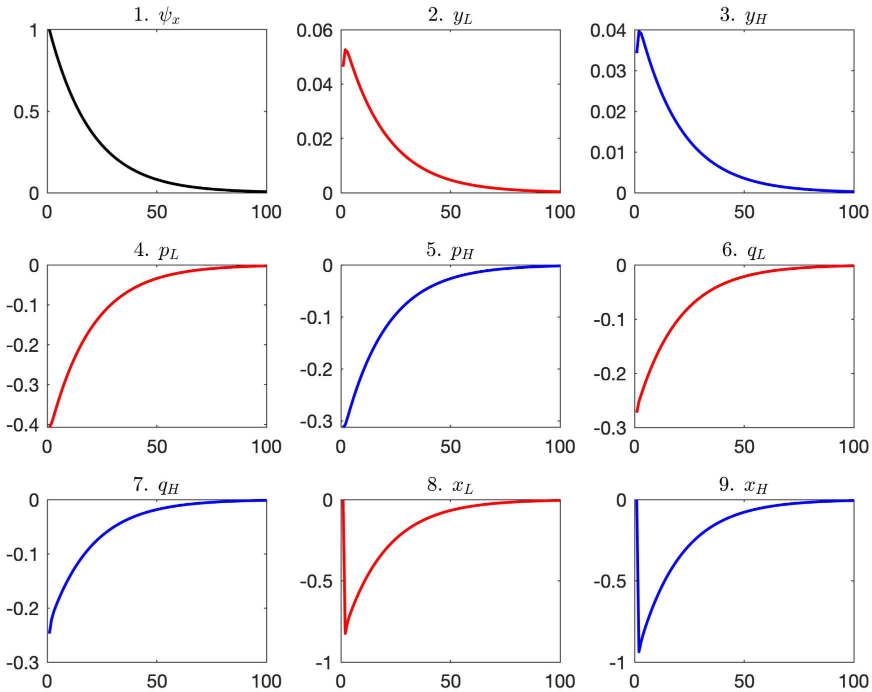


Figure 41: Impulse Response Functions for the selected variables following a positive shock to ψ_x.

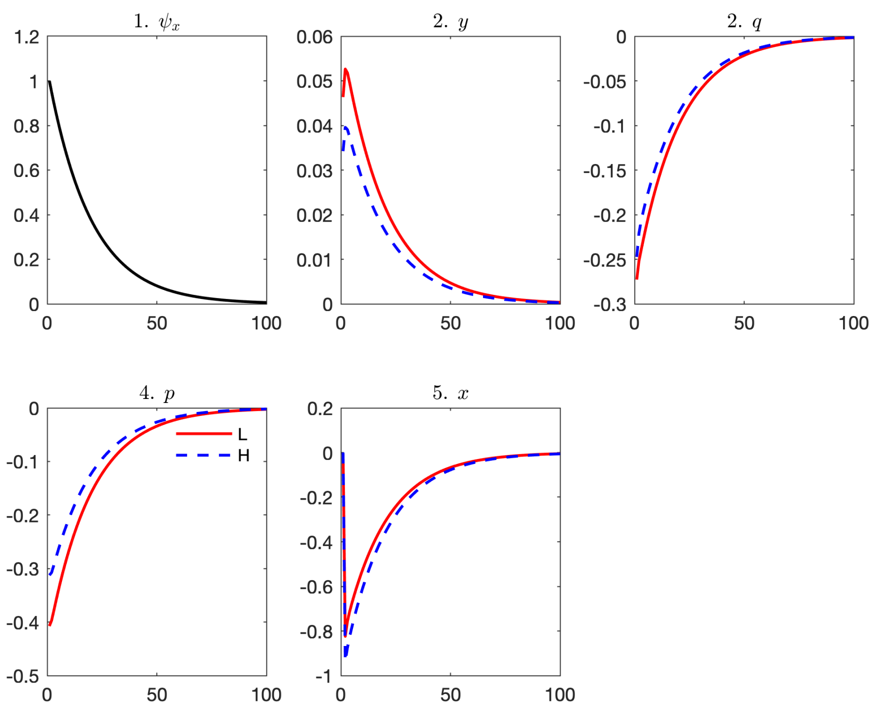


Figure 42: Comparison of Impulse Response Functions for selected variables following a positive shock in ψ_x.

Notice that each panel reports the proportional change for the variable under consideration, in percentage points. For instance, Panel 1 reports a persistent increase in ψ_x for 100 periods, after an increase of 1 % on impact, except for the inventories, for which we compute the absolute variation.

Below we show the same results for a persistent fall in β for 100 periods, after a fall of 1 % on impact.

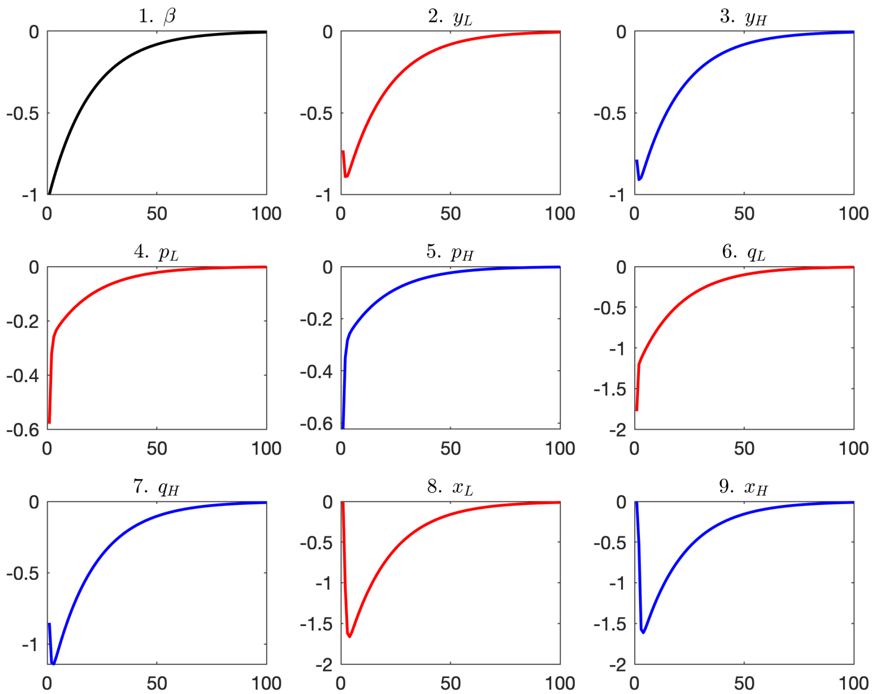


Figure 43: Impulse Response Functions for the selected variables following a negative shock to β.

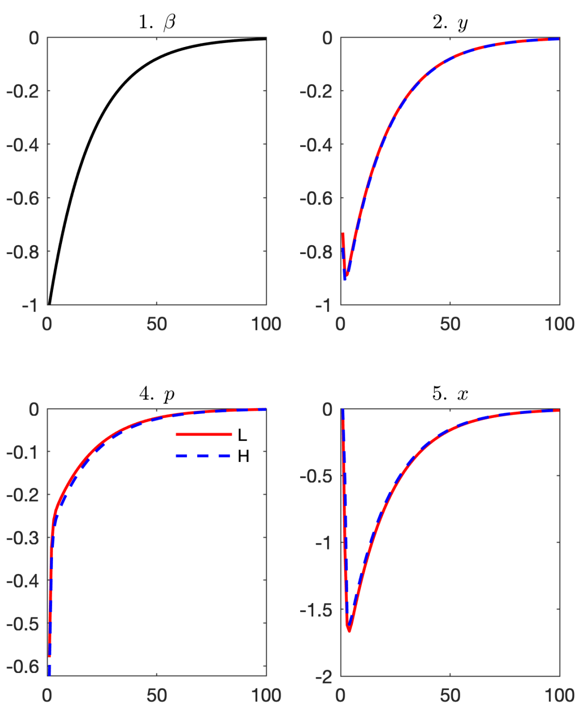


Figure 44: Comparison of Impulse Response Functions for selected variables following a negative shock in in β.

The relationship between inventory holding costs, represented by ψ_x, and production levels

(y_L in low demand, y_H in high demand) can be understood through inventory management and production planning strategies.

When ψ_x is low, the cost of holding inventory is minimal. Firms tend to maintain larger inventories, which keeps prices high and demand lower. As inventory costs rise, firms lower prices to clear out excess stock, but not enough to significantly boost demand. Consequently, they reduce production and rely more on existing inventory to meet sales, resulting in steady production levels with minimal adjustments to demand fluctuations.

In contrast, when ψ_x is high, holding inventory becomes costly. Firms reduce inventory levels and adopt a just-in-time production approach, producing goods closer to the time they are needed. This increased production flexibility allows them to meet demand directly without maintaining large inventories. Lower prices in this scenario stimulate higher demand, prompting firms to boost production to manage the increased demand effectively.

Firms balance the trade-off between inventory holding costs and production costs. With low ψ_x, the emphasis is on holding more inventory and maintaining steady production. With high ψ_x, the focus shifts to minimizing inventory and increasing production flexibility.

For instance, a retailer with fluctuating demand will keep large stocks and produce less frequently when inventory costs are low. Conversely, when inventory costs are high, the retailer minimizes stock and produces more frequently to avoid high holding costs.

Overall, changes in ψ_x lead to different strategies for managing production and inventory, resulting in a non-linear relationship between ψ_x and production levels (y_L and y_H).

H Steady State Quantitative Model

Discretization of the model. We define the following aggregate variables: Aggregate Price (P), Aggregate Labor (L), Aggregate Sales (S), Inventories (X), Quantity (Q), Aggregate Profits (Θ), Consumption (C), and the Pricing Kernel (λ).

Using these, we define the vector $\boldsymbol{X} = \{\boldsymbol{P^0}, \boldsymbol{L^0}, \boldsymbol{S}, P, L, S, X, Q, \Theta, C, \lambda\}$. The distribution of firms is discretized using a histogram $\boldsymbol{S}$, as described in Young (2010), with $\boldsymbol{S'} = \Pi_{\varepsilon\times\varepsilon}\boldsymbol{S}$, where $\Pi_{\varepsilon\times\varepsilon}$ represents the transition matrix implied by the decision rules.

First, we discretize the policy rules (p^0, l^0) using the vectors P^0 and L^0, respectively. These policy rules must satisfy the Euler equations for each q and ε, or meet the relevant constraints.

Thus, the equilibrium conditions must exhibit dynamics such that:

$$F(X', X, \eta', z') = 0,$$

where η is a vector of expectation errors. By expressing the system in this form, we know how to solve it. Note that the approximate equilibrium conditions form a system of $3n_q n_\varepsilon + 8$ equations, where n_q is the number of grid points for quantity, n_ε is the number of grid points for demand shocks, and the number 8 corresponds to our aggregate variables.

The expectation errors arise from the aggregate productivity shock, and the expectation operator $\mathbb{E}$ can be rewritten as:

$$\mathbb{E}_{S_{-1},Z,\Lambda}(x) = x + \eta.$$

We have as many expectation operators as we have Euler equations. Finally, we can replace the aggregate state (Z, Λ) with $(Z, \boldsymbol{S})$ and compute the recursive competitive equilibrium.

Computing the recursive competitive equilibrium. Our goal is to solve:

$$F(X^*, X^*, 0, 0) = 0.$$

In order to solve for the steady state, we adopt the following algorithm. The problem consists of solving a system of equations. The steady state is solved using 10 idiosyncratic demand shocks and 100 points in the quantity grid.

First, note that the demand each firm faces is given by:

$$S(q, \varepsilon; Z, \Lambda) = \frac{1}{p(q, \varepsilon; Z, \Lambda)^{\gamma}} \varepsilon P(Z, \Lambda)^{\gamma} S(Z, \Lambda).$$

The algorithm to find the steady-state allocation consists of solving a root-finding problem in the wage $w(Z, \Lambda)$ and the total sales $S(Z, \Lambda)$. In other words, our goal is to find a wage that equalizes labor supply and demand, and a value for $S(Z, \Lambda)$ such that, when we solve the problem for individual firms and aggregate their optimal results, the sum of the individual firms' optimal sales equals $S(Z, \Lambda)$.

To solve for the stationary recursive competitive equilibrium, we set $\Pi = 1$, $Z = 1$, and we assume that the final good is the numeraire in this economy, meaning the price will be such that $P(Z, \Lambda) = 1$.

The algorithm is as follows:

1. First, guess a value for w and S. Solve the individual firm's problem as defined in equations (58) - (61), using Value Function Iteration or the Endogenous Grid Method.
2. Using the policy rules, determine the stationary distribution by iterating on the law of motion.
3. Determine the labor demand for each level of quantity and idiosyncratic demand shocks. Then, aggregate labor demand using the stationary distribution of firms over quantity and idiosyncratic demand shocks to obtain:

$$\int l(q, \varepsilon; Z, \Lambda)\, d\varepsilon\, d\Lambda(q \times \varepsilon).$$

4. Determine the price chosen for each level of quantity and idiosyncratic demand shocks. Then,

aggregate the price using:

$$P(Z,\Lambda) = \left(\int p(q,\varepsilon;Z,\Lambda)^{1-\gamma} \varepsilon \, d\varepsilon \, d\Lambda(q \times \varepsilon) \right)^{\frac{1}{1-\gamma}}.$$

5. Finally, calculate the following aggregate:

$$S(Z,\Lambda) = \left(\int_0^1 S(q,\varepsilon;Z,\Lambda)^{\frac{\gamma-1}{\gamma}} \, d\varepsilon \, d\Lambda(q \times \varepsilon) \right)^{\frac{\gamma}{\gamma-1}}.$$

6. Using the value of consumption:

$$C(Z,\Lambda) = \left(1 - \frac{\kappa}{2}(\Pi(Z,\Lambda)-1)^2\right) S(Z,\Lambda) - \frac{\psi_x}{2} \int x(q,\varepsilon;Z,\Lambda)^2 \, d\varepsilon \, d\Lambda(q \times \varepsilon),$$

calculate labor supply as:

$$L(Z,\Lambda) = \left(\frac{w(Z,\Lambda)}{\chi C(Z,\Lambda)} \right)^{\phi}.$$

7. The goal is to solve for a wage w^* such that:

$$\int l(q,\varepsilon;Z,\Lambda) \, d\varepsilon \, d\Lambda(q \times \varepsilon) = L(Z,\Lambda),$$

and a value S^* such that $P(Z,\Lambda) = 1$.

8. We solve for these two values using a root-finding algorithm.

Linearization. The function $F(X', X, \eta', z')$ is numerically differentiated around the non-stochastic steady state $X = X' = X^*$, resulting in the following partial derivatives:

$$F_1 = \left(\frac{\partial F}{\partial X'} \right)_{X=X^*},$$

$$F_2 = \left(\frac{\partial F}{\partial X} \right)_{X=X^*},$$

$$F_3 = \left(\frac{\partial F}{\partial \eta'} \right)_{X=X^*},$$

$$F_4 = \left(\frac{\partial F}{\partial z'} \right)_{X=X^*}.$$

Therefore, the equation can be written as:

$$F_1(X' - X^*) + F_2(X - X^*) + F_3\eta' + F_4 z' = 0.$$

The system can be reformulated using the methodology in Sims (2002) as:

$$\Lambda_0 y' = \Lambda_1 y + C + \psi z' + \phi \eta',$$

where $y' = X' - X^*$, $\Lambda_0 = -F_1$, $\Lambda_1 = -F_2$, $C = 0$, $\psi = F_3$, and $\phi = F_4$. The outcome of the method in Sims (2002) are the matrices A and B such that:

$$y' = Ay + Bz'.$$

The linearization and perturbation methods are automatically handled by Dynare (Adjemian et al. (2011)). Dynare computes the partial derivatives and then solves the system.

Below, we present the steady-state values of the individual decisions of firms along the quantity and demand shocks schedule:

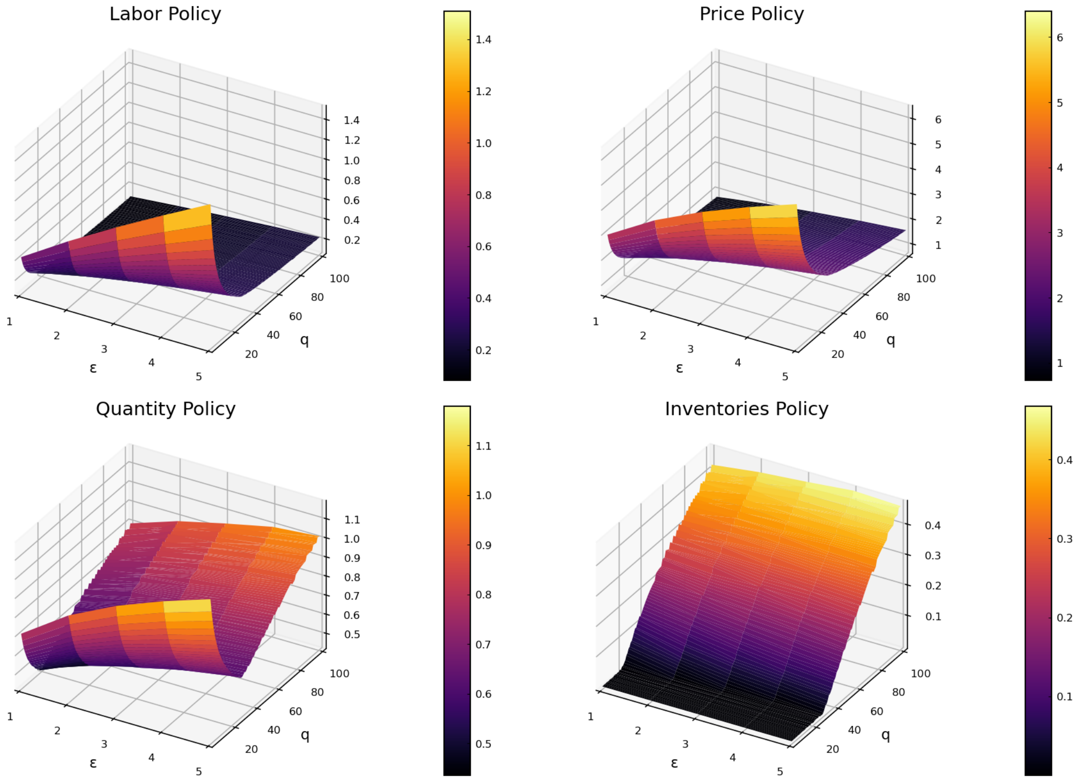


Figure 45: Policy functions of individual firm's problem.

I Preference Shocks in β

I.1 Dynamics for β with Interest rate shock

In Figure 46, we show the equilibrium conditions for the selected variables following a negative shock in β.

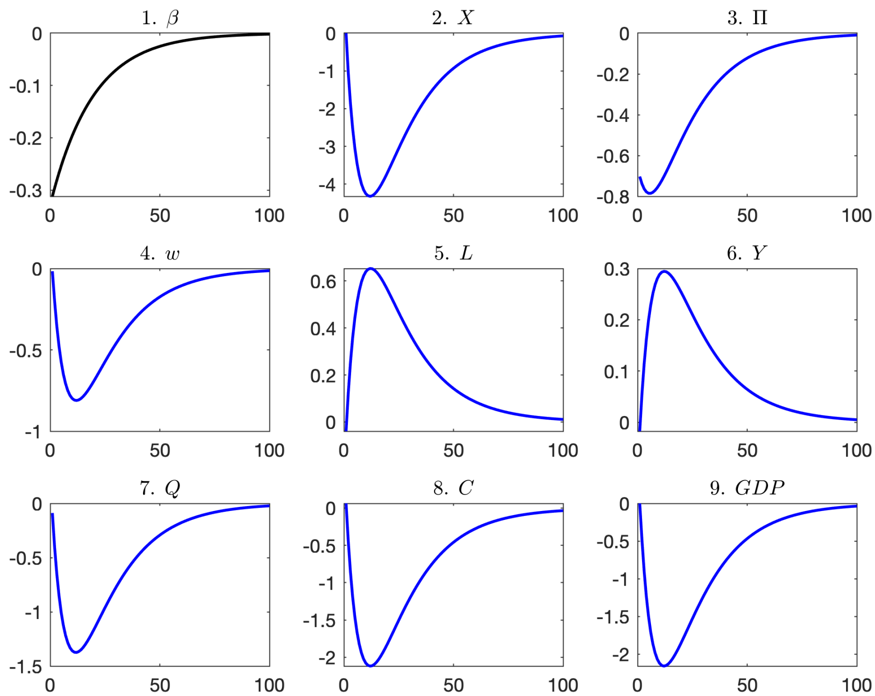


Figure 46: Impulse Response Functions for the selected variables following a negative shock to β.

I.2 Dynamics for β with Monetary Policy

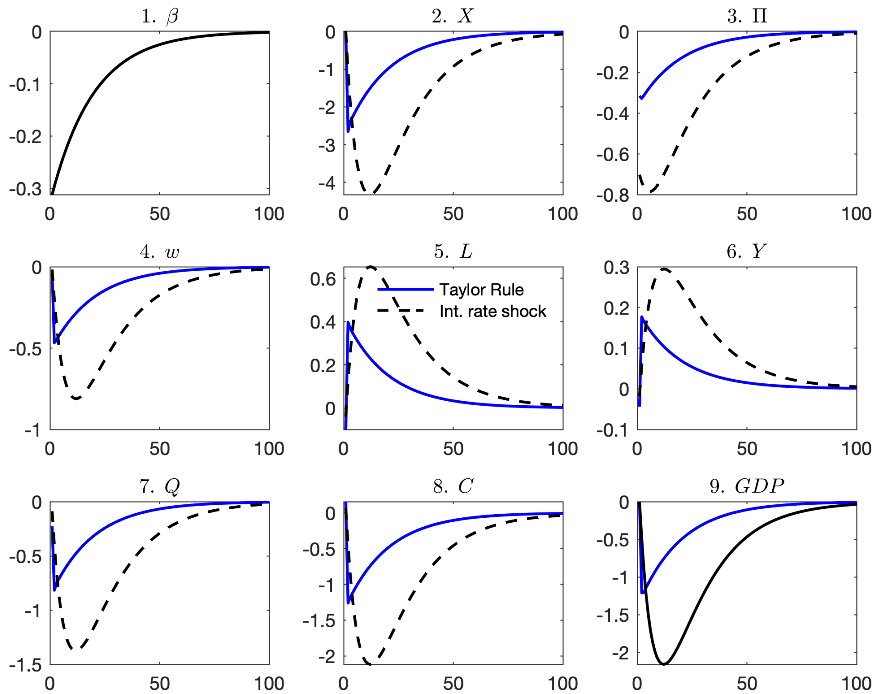


Figure 47: Impulse Response Functions for the selected variables following a negative shock in β.

J Monetary Policy and Shocks in ψ_x and Z

We now examine an economy where monetary policy is implemented via a Taylor rule as in

$$\frac{R_t^N}{R^N} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}, \tag{140}$$

where R^N is the steady-state nominal interest rate, Π is the steady-state inflation rate, and ϕ_Π is the response of the nominal interest rate to inflation.

First, we compare two scenarios: one where the Central Bank adopts a policy rule according to the Taylor Rule with $\phi_\Pi = 1.5$ as in Galí (2015) and reacts to deviations from the steady-state inflation rate set at $\Pi_t = 1$, and another where there is a pesisent shock to the nominal interest rate as discussed in Section 5.2.

The goal is to assess the effects of the endogenous response of monetary policy and to understand the model dynamics under different inflation paths. When the Taylor Rule is applied, as expected, all selected variables exhibit lower volatility and return more quickly to their steady-state values. In particular, the Taylor Rule helps stabilize inflation, preventing it from becoming excessively volatile.

Following a positive shock in ψ_x, the level of inventories decreases, but the reduction in prices is less pronounced compared to the scenario where agents can anticipate the path of interest rate.

This outcome highlights the Taylor Rule's role in minimizing deviations from steady-state values, thereby moderating the impact on prices.

Those results are depicted in Figure 48 for a positive shock in ψ_x.[26]

		$\boldsymbol{\psi_x}$ $\boldsymbol{\phi_\Pi = 1.5}$	$\boldsymbol{Z}$ $\boldsymbol{\phi_\Pi = 1.5}$
w	Mean	1.000	1.006
	Std	0.001	0.017
Π	Mean	1.001	1.000
	Std	0.159	0.042
X	Mean	0.318	0.322
	Std	0.012	0.047
C	Mean	1.585	1.602
	Std	0.006	0.036
L	Mean	0.262	0.261
	Std	0.002	0.009
Q	Mean	0.876	0.882
	Std	0.004	0.020
Y	Mean	0.558	0.560
	Std	0.001	0.006
GDP	Mean	1.619	1.636
	Std	0.006	0.035
		Correlations	
corr(X, Π)		0.9585	-0.9321
corr(X, X_{-1})		0.9452	0.8953
corr(GDP, GDP_{-1})		0.9532	0.9670

Table 10: First and second moments for key variables are provided for the following scenarios: a shock to ψ_x with a Taylor rule ($\phi_\Pi = 1.5$) and a shock to Z with a Taylor rule ($\phi_\Pi = 1.5$).

[26] Similar results are found when we apply a negative shock in β as we can see in Appendix I.2

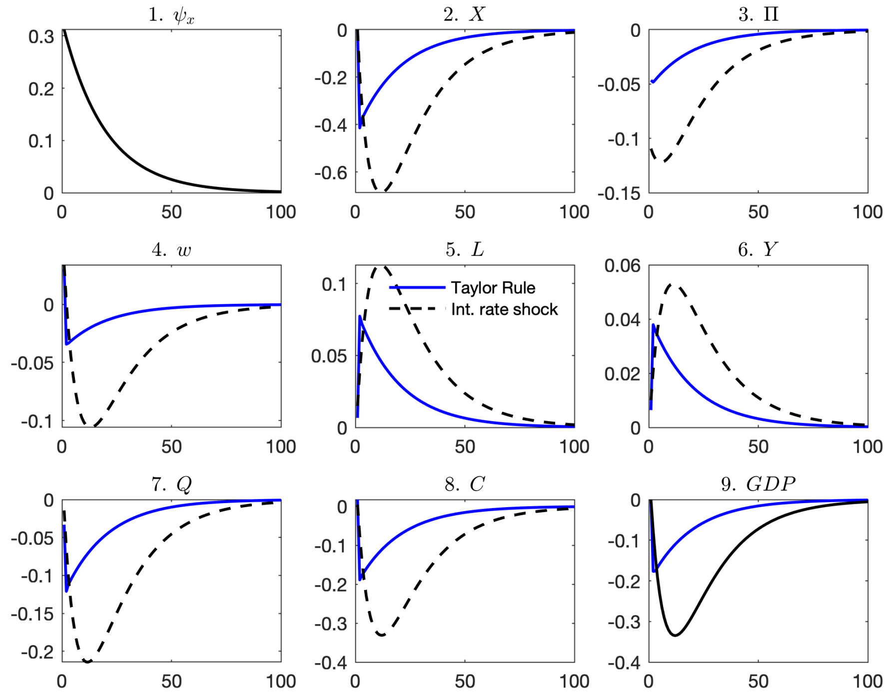


Figure 48: Impulse Response Functions for selected variables following a positive shock in ψ_x.

We now compare the dynamics of the economy under a negative TFP shock with and without the implementation of the Taylor rule. The results are shown in Figure 49. While the introduction of monetary policy reduces volatility across most variables, it does not mitigate inflation, which actually increases more compared to a scenario without monetary policy intervention.

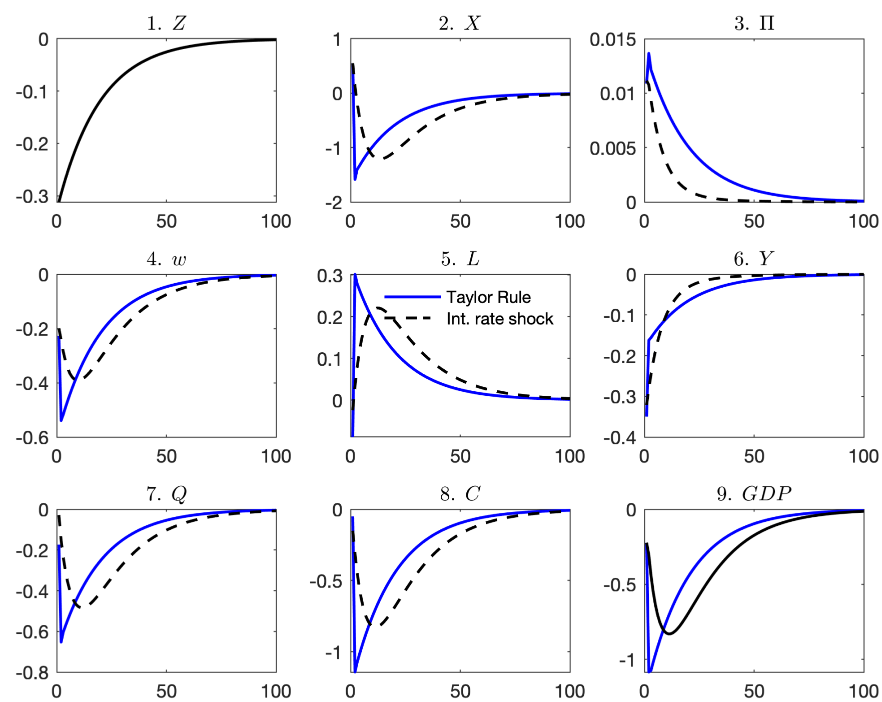


Figure 49: Impulse Response Functions for selected variables following a negative shock in Z.

These findings are confirmed by the second-order moments presented in columns (1) and (2) of Table 10.

Finally, we compare two distinct situations: one with a low level of inventories and another with a higher level after a positive shock in ψ_x. We observe that, under the Taylor rule, the decrease in prices is similar across both inventory levels. This outcome occurs because the Taylor rule aims to minimize deviations from the steady-state inflation rate. Consequently, the paths of the selected variables are similar in both cases, as the Taylor rule effectively moderates the impact of monetary policy shocks. Figure 50 shows the IRFs.

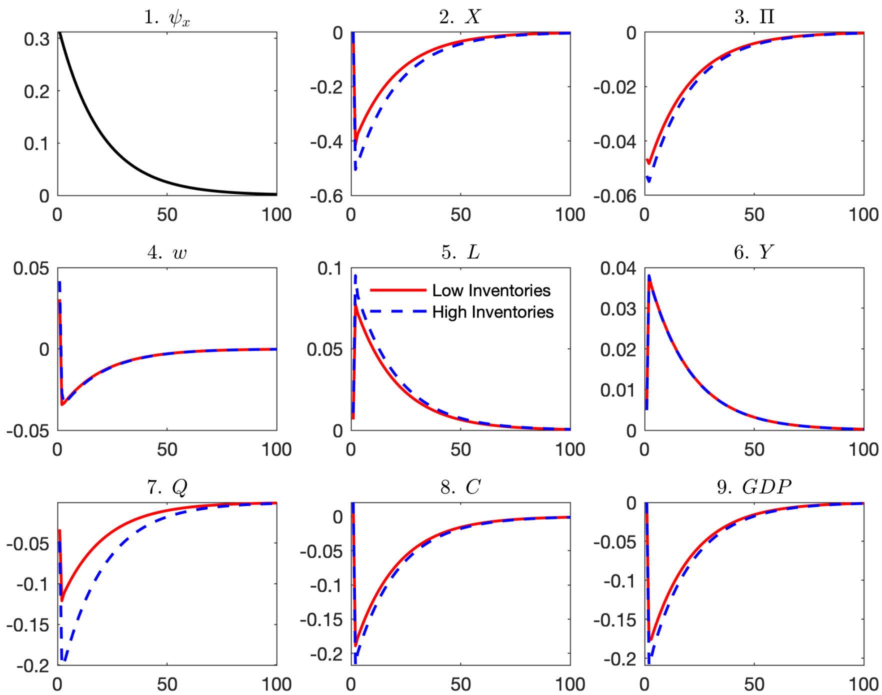


Figure 50: Impulse Response Functions for selected variables following a positive shock in ψ_x with a Taylor Rule of $\phi_\Pi = 1.5$, compared for two distinct levels of inventories.

In order to understand the model dynamics when inflation follows a different path, we examine two inventory levels, higher and lower, in a scenario where the Taylor rule coefficient is set to $\phi_\Pi = 1.01$. This represents a situation where the endogenous response of monetary policy is weaker. The IRFs for this case are shown in Figure 51. By comparing Panel 3 of Figure 51 with Panel 3 of Figure 50, we observe that a lower Taylor coefficient results in a greater divergence between the two inflation paths. This indicates that the less hawkish the Central Bank, the more inventories influence the inflation trajectory. Consequently, the cost of carrying inventories will have a larger effect in decreasing inflation.

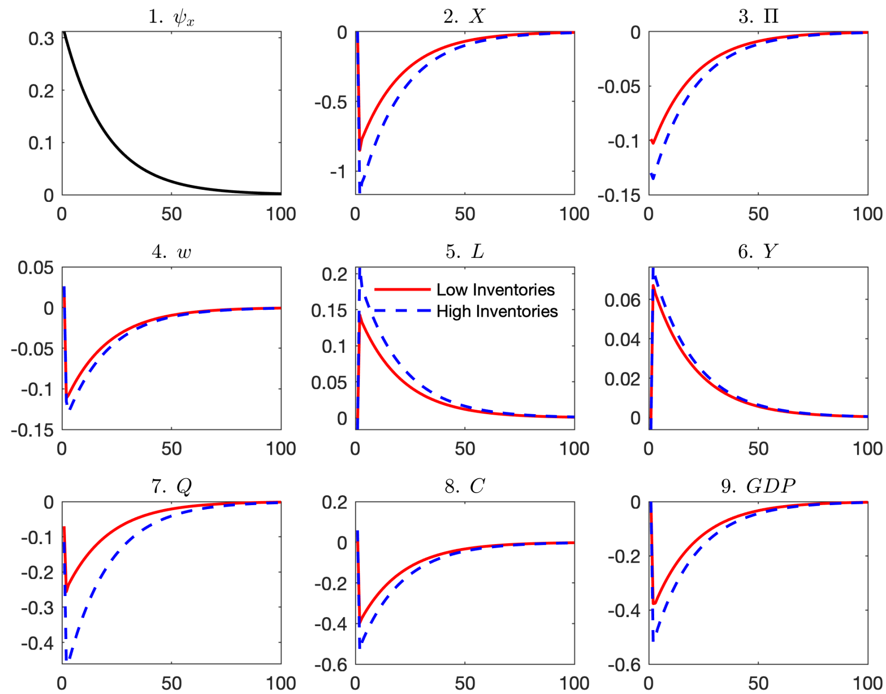


Figure 51: Impulse Response Functions for selected variables following a positive shock in ψ_x with a Taylor Rule of $\phi_\Pi = 1.01$, compared for two distinct levels of inventories.

This occurs because the model already incorporates a natural mechanism through firms' price-setting decisions that helps reduce inflation. This result suggests that monetary contractions reduce inflation more effectively when inventory levels are high, implying that central banks may not need to pursue overly aggressive tightening when inventory levels are elevated.[27]

K TFP Shock

We compare two scenarios following a negative shock in Z: one with low inventory levels and another with higher inventory levels. We observe that, under the Taylor rule, prices increase more when inventories are lower. This suggests that a negative TFP shock heightens the need to boost inventories, and the lower the inventory level, the higher the prices required to achieve this. Consequently, under such a shock, the Taylor rule seems less effective in moderating inflation, with inventories playing a more crucial role in stabilizing the economy. The Impulse Response Functions (IRFs) illustrating these dynamics are shown in Figure 52.

[27] In Appendix K we show the same exercise for the case where we have a negative shock in Z.

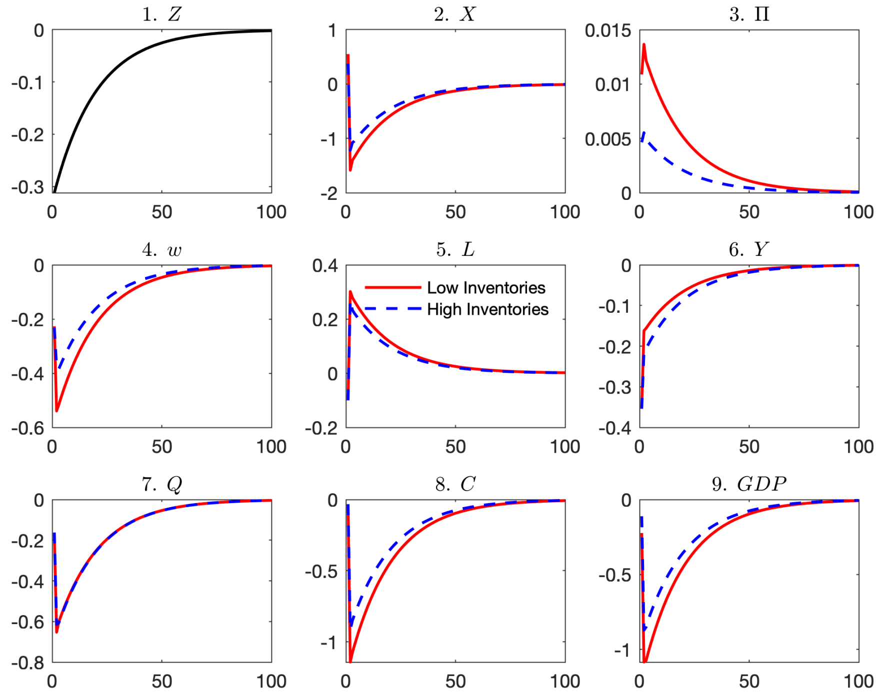


Figure 52: Impulse Response Functions for selected variables following a negative shock in Z with a Taylor Rule of $\phi_\Pi = 1.5$, compared for two distinct levels of inventories.

To analyze how inflation dynamics change under different conditions, we examine two inventory levels—high and low—while setting the Taylor rule coefficient to $\phi_\Pi = 1.01$. This coefficient represents a weaker endogenous response of monetary policy. The IRFs for this scenario are shown in Figure 53. By comparing Panel 3 of this figure with Panel 3 of Figure 52, we observe that a lower Taylor coefficient leads to similar inflation paths. This suggests that the Taylor rule's effectiveness as a stabilization mechanism diminishes under a TFP shock when inventories are considered.

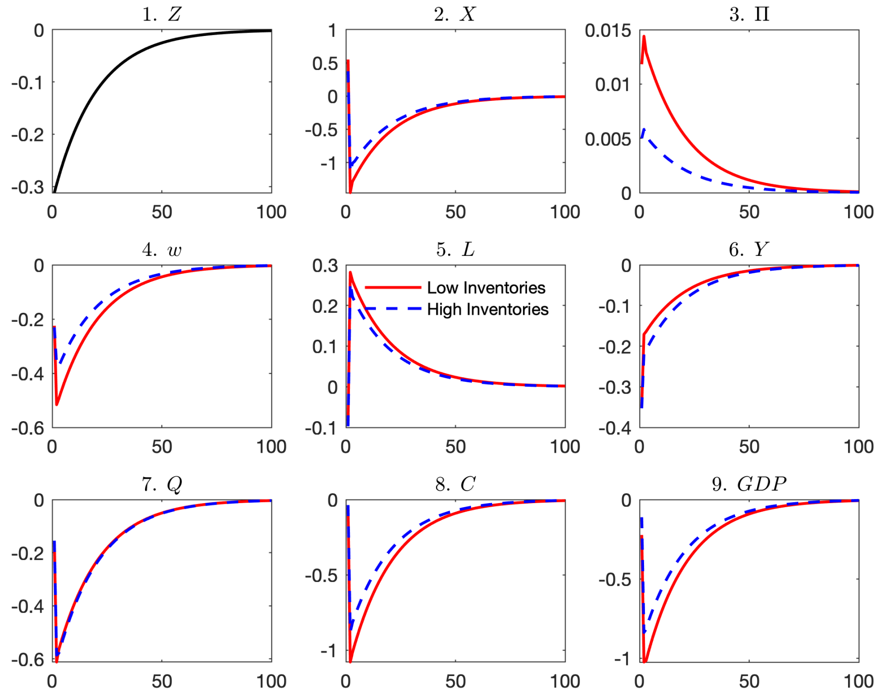


Figure 53: Impulse Response Functions for selected variables following a positive shock in ψ_x with a Taylor Rule of $\phi_\Pi = 1.01$, compared for two distinct levels of inventories.

L Empirical evidence from the housing market

L.1 Constructing a housing inventory metric

Our empirical exercise in Section 6 investigates whether the response of the cost of housing (to monetary policy shocks) differs depending on the size of the housing inventory "INV_t", the fraction of homes that is not being occupied. The US Census Bureau produces two series which are getting at this concept: one for rental properties (FRED code: RRVRUSQ156N) and one for owner-occupied properties (FRED code: RHVRUSQ156N). These two series are highly correlated (with a correlation coefficient of 0.74), which is intuitive. We proceed by combining these two series into a single "home vacancy rate", which is constructed as a weighted-average between the two – with the weight determined by the homeownership rate (FRED code: RSAHORUSQ156S). Finally, since the original series are only available at the quarterly frequency, we use linear interpolation to obtain a monthly series. Given the high degree of persistence in the quarterly series (an autocorrelation coefficient of 0.96 at the quarterly frequency), this is unlikely to be a major issue.

L.2 IRFs for standard variables

As mentioned in the main text, the Bauer-Swanson monetary policy shock series produces intuitive responses in core variables – building confidence that the series captures true monetary policy shocks. Figures 54 - 57 show IRFs that follow from estimating the equivalent of (68) for different dependent variables Y:

$$\Delta^h Y_{t+h} = \alpha_h + \beta_h MPS_t + \delta_h X_t + \epsilon_{t,h}, \tag{141}$$

where the dependent variable Y enters (141) in natural logs, except for the rate of unemployment (for which outcomes are therefore in percentage points). To be consistent with the analyses featured in the main body of the text, we control for lags of the monetary policy shock in X_t – so that the righ- thand side of (141) always contains 12 months worth of (lagged) MPS-realizations.

First, Figure 54 captures the response of the CPI (FRED code: CPIAUCSL), which aligns with conventional prior notions of what the response should look like in response to a monetary policy shock; the same can be said for the response of the industrial production index (FRED code: INDPRO) in Figure 55 and the rate of unemployment in Figure 56 (although the point estimate is never significantly different from zero; however, by performing a joint-test a la Inoue et al. (2023), we are able to reject the hypothesis that monetary policy shocks have no impact on unemployment at the 1% significance level). Finally, Figure 57 depicts the response of the US equity index (FRED code: SPASTT01USM661N, which is for the S&P500 index). Here, it is comforting to see that the entire impact gets priced in quickly, leading to a flat profile of the IRF as time passes (consistent with the Efficient Market Hypothesis).

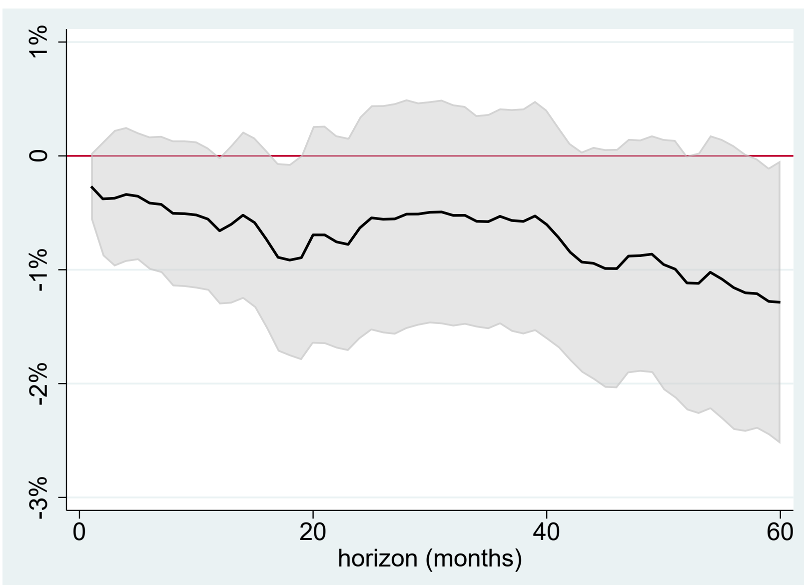


Figure 54: Response of US CPI to a 25-bp contractionary monetary policy shock, estimated via equation (141). Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

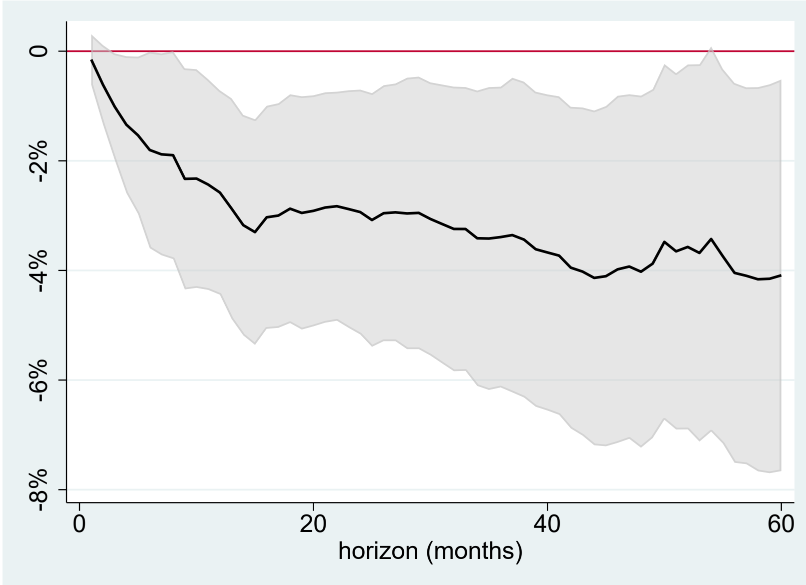


Figure 55: Response of US industrial production to a 25-bp contractionary monetary policy shock, estimated via equation (141). Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

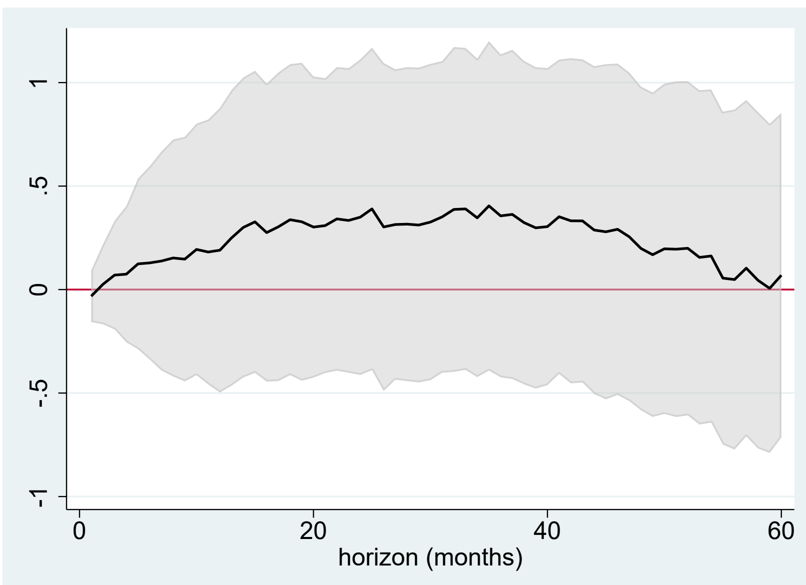


Figure 56: Response of US unemployment rate to a 25-bp contractionary monetary policy shock, estimated via equation (141). Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

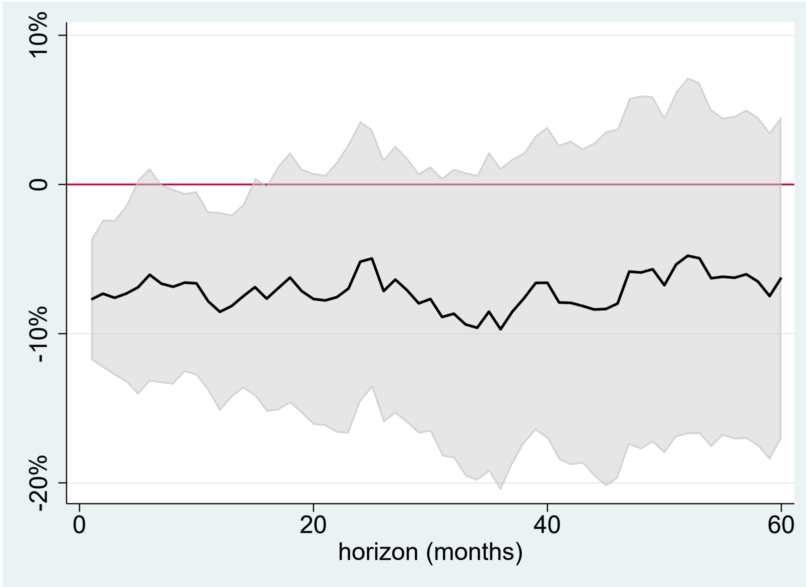


Figure 57: Response of US equity index to a 25-bp contractionary monetary policy shock, estimated via equation (141). Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

L.3 Robustness of main result

Our main finding, that the cost of housing is more sensitive to monetary policy shocks when the home vacancy rate is higher, is very robust. Here, we document some of the robustness exercises we have conducted.

First, our main result is robust to controlling for the state of the business cycle (as proxied by the rate of unemployment) within the controls vector X_t in equation (67); see Figure 58. This suggests that the effect we are picking up is not driven by variations in the state of the business cycle.

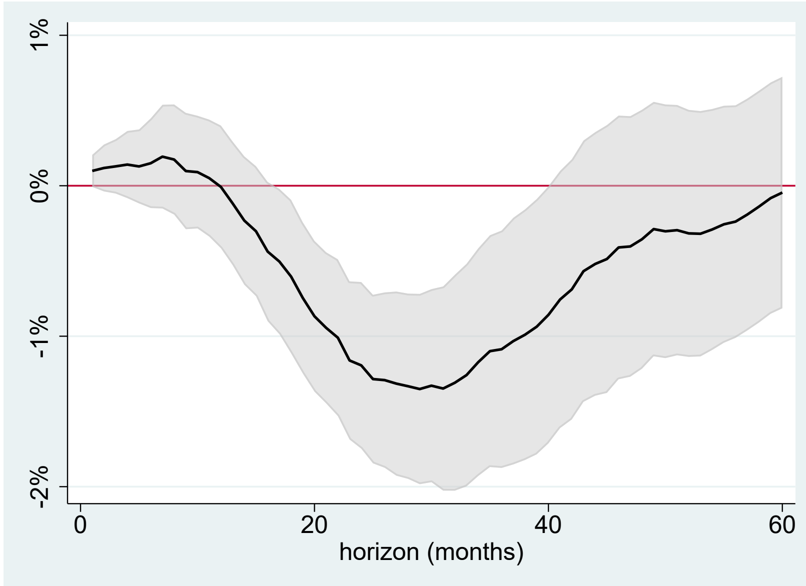


Figure 58: Additional response of CPI OER to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67) with the rate of unemployment added to the vector of controls (X_t). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

Along similar lines, a very similar IRF emerges when adding 12 monthly lags of the dependent variable "HC", the CPI OER series, to the vector of controls X_t (Figure 59) or when enriching X_t with 12 monthly lags of the overall CPI alongside industrial production (Figure 60). The fact that adding these controls does relatively little to subsequent findings confirms the notion that the underlying Bauer-Swanson series is well-orthogonalized with respect to available data – cutting the need to include covariates.

Replicating our baseline analysis when using the housing-related component of the PCE index as dependent variable produces an even stronger result (in the sense of being larger in magnitude and more persistent; see Figure 61). At the same time, our core result also shows up when using the overall CPI as dependent variable (Figure 62), which is perhaps not that surprising given OER accounting for about one-quarter of the aggregate.

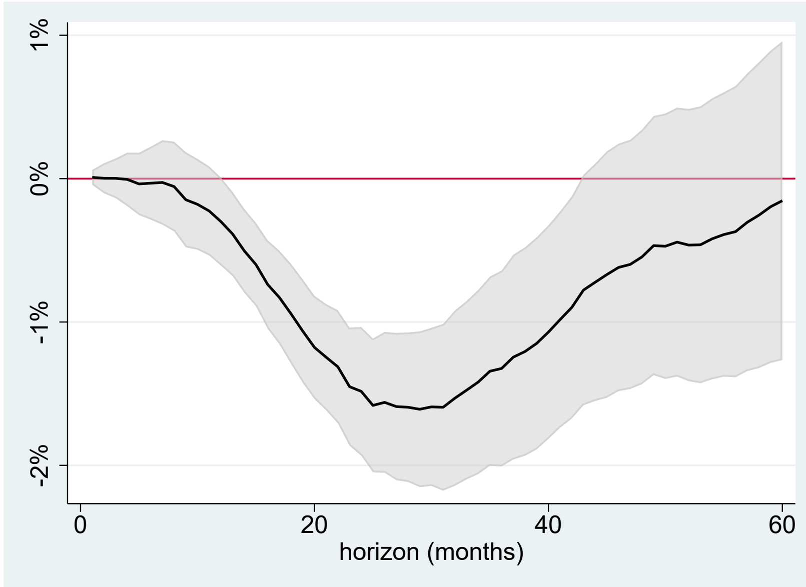


Figure 59: Additional response of CPI OER to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67) with lags of CPI OER added to the vector of controls (X_t). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

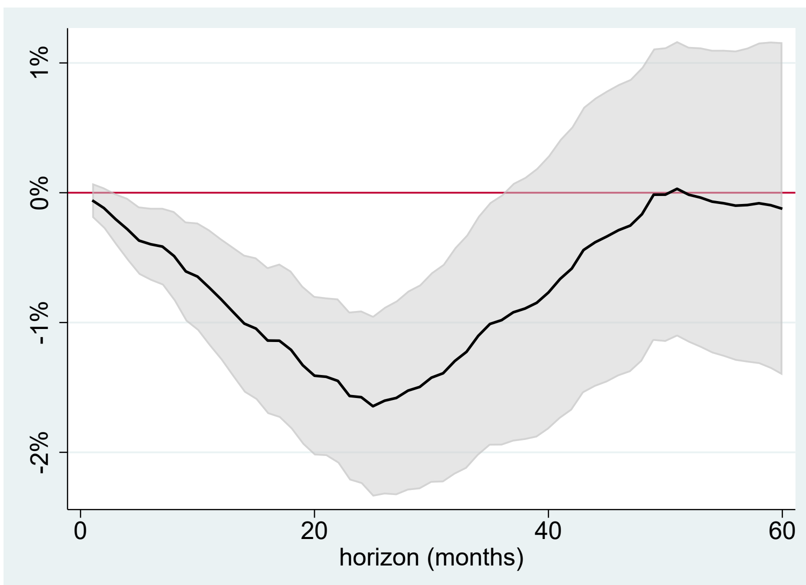


Figure 60: Additional response of CPI OER to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67) with lags of overall CPI and industrial production added to the vector of controls (X_t). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

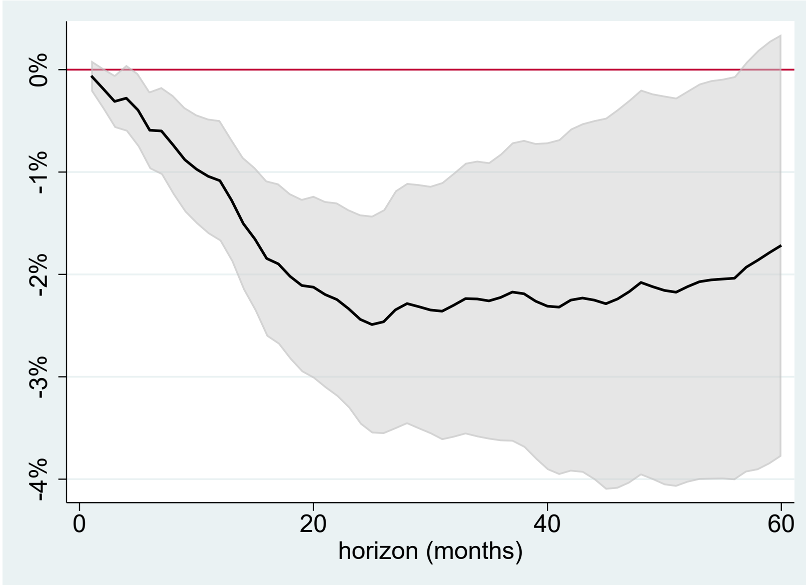


Figure 61: Additional response of the housing component of the PCE index to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

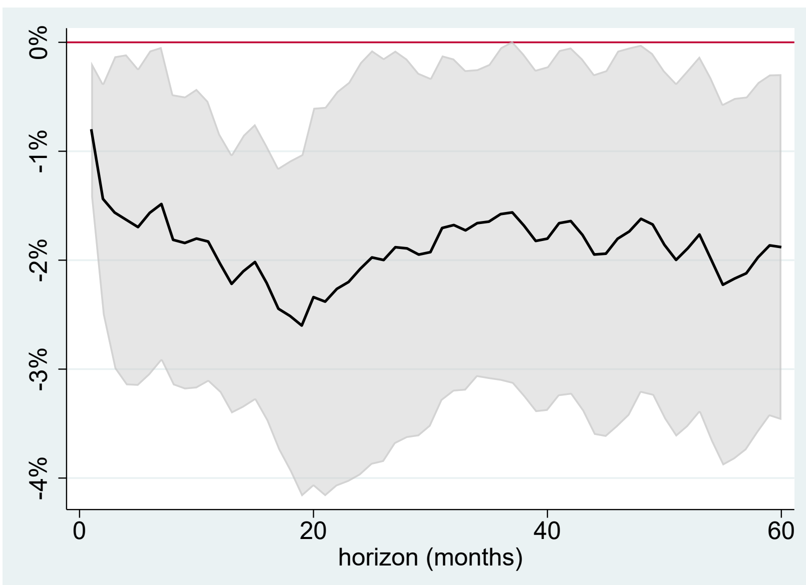


Figure 62: Additional response of CPI to a 25-bp contractionary monetary policy shock, due to a 1-pp increase in the home vacancy rate, estimated via equation (67). The figure plots $\hat{\gamma}_h$. Shaded area represents the 90% confidence interval, calculated via Newey-West standard errors.

Our main result also obtains when approaching the question via a different method. Next to considering an interaction effect (as done in the main text), one could also follow Auerbach & Gorodnichenko (2012) and Tenreyro & Thwaites (2016) by carrying out a state-dependent analysis. In that case, we estimate:

$$\Delta^h \ln HC_{t+h} = F(INV_t)\left[\overline{\alpha_h} + \overline{\beta_h} MPS_t + \overline{\delta_h} X_t\right], \qquad (142)$$
$$+ \left[1 - F(INV_t)\right]\left[\underline{\alpha_h} + \underline{\beta_h} MPS_t + \underline{\delta_h} X_t\right] + \epsilon_{t,h}$$

where overlines (underlines) indicate coefficient estimates when the housing vacancy rate "INV_t" is high (low). To maintain consistency with our exercise in the main text, the vector of controls continues to include a year worth of monthly lags of the monetary policy shock. The probability of being in the high regime is calculated via the logistic function:

$$F(INV_t) = \frac{\exp\left(\theta(INV_t - c)/\sigma_{INV}\right)}{1 + \exp\left(\theta(INV_t - c)/\sigma_{INV}\right)},$$

where we take c to be the median of INV_t over our sample period, while we pick a standard value for $\theta = 1.5$ (which governs the intensity of regime switching); σ_{INV} represents the standard deviation of INV_t.

Figure 63 plots the results of this exercise. The coefficient estimates $\hat{\overline{\beta_h}}$ and $\underline{\hat{\beta}_h}$, depicted in the figure, trace out the average IRFs to monetary policy shocks depending on whether the housing vacancy rate was "high" (i.e., $F(INV_t) \approx 1$) or "low" (i.e., $F(INV_t) \approx 0$) at the time of the shock. As one can see, this exercise supports the conclusion that stems from the interactive regression reported in the main text: contractionary monetary policy shocks do more to lower the cost of housing when the housing vacancy rate stands at a higher level. In fact, when the housing market is tight (low INV_t), landlords seem able to pass-on increases in their cost of borrowing – which would only add to inflation (potentially generating a "price puzzle" in the aggregate).

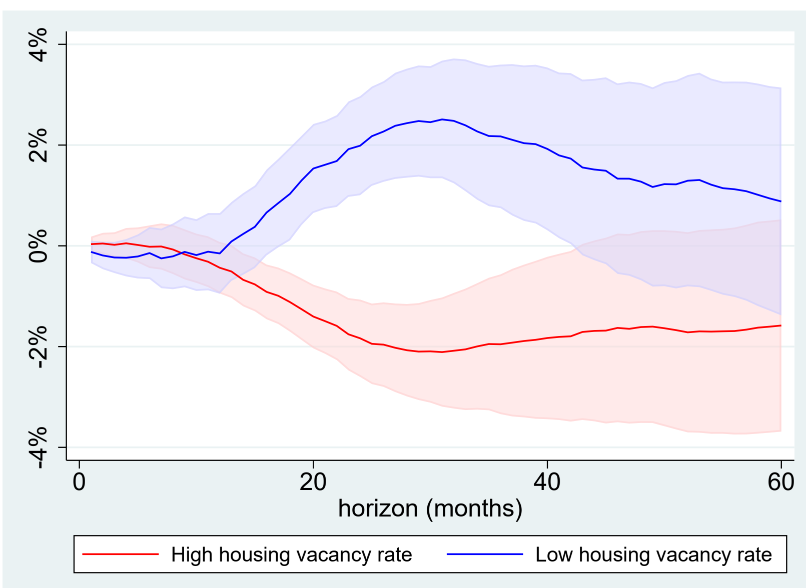


Figure 63: Responses of CPI OER to a 25-bp contractionary monetary policy shock, estimated via equation (142). The figure plots $\hat{\overline{\beta_h}}$ and $\underline{\hat{\beta}_h}$. Shaded areas represent the 90% confidence intervals, calculated via Newey-West standard errors.