Commodity Prices and Sovereign Risk*

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Abstract

Swings in global commodity prices are often associated with higher sovereign risk. Using a shift-share strategy, we show that import-price shocks raise sovereign spreads and reduce borrowing, while export-price shocks have smaller effects and tend to lower spreads. To explain these patterns, we develop a sovereign default model in which a common factor drives global commodity prices. Global price shocks affect sovereign risk through two channels: a *price-index channel*, which raises consumption costs, and a *terms-of-trade income channel*, which shifts real income. Each country is affected differently depending on its trade position. For importers of goods driving global prices, a surge in global prices raises consumption costs more than real income, increasing default incentives and sovereign spreads.

JEL-classification: F34, F41, E32, E44, F62.
Key words: Sovereign Default, Commodity Prices, Terms-of-Trade, External Borrowing, Global Shocks, Spreads.

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1 Introduction

Sovereign defaults often occur in waves that coincide with swings in commodity prices. Exporters benefit from commodity booms through improved terms of trade, relaxed borrowing constraints, and falling spreads, whereas import-dependent economies face the opposite dynamics. The effect of global price movements on aggregate consumption depends on a country's trade position. A rise in export prices raises real income and strengthens repayment incentives, while an increase in import prices acts as a negative income shock that worsens default risk.

We test these mechanisms empirically using a shift–share design that exploits cross-country differences in import and export baskets. We construct country-specific price shocks by combining global commodity-price innovations with lagged trade weights. The evidence confirms a strong asymmetry: import-price shocks raise sovereign spreads and reduce external borrowing, while export-price shocks compress spreads and have weaker effects on borrowing. These results align closely with the idea that terms-of-trade fluctuations act as real income shocks for countries with different trade exposures.

Motivated by these findings, we develop a quantitative sovereign default model that accounts for this heterogeneity by introducing *global price shocks* as the central source of risk. In contrast to income-based models such as Arellano (2008), where shocks shift output directly, here they operate through two structural channels tied to observables: a *price-index channel*, which captures how global price movements change the cost of the consumption basket, and a *terms-of-trade income channel*, which captures how the same global movements affect the income generated by trade.

The *price-index channel* reflects a cost-of-living effect: when the world prices of highly traded goods increase, the domestic consumption basket becomes more expensive. For a country that primarily imports the goods that move with the global factor, such as oil or mineral commodities, this channel tightens repayment capacity by eroding real income and raising default risk. The *terms-of-trade income channel*, by contrast, reflects an income effect: when a country exports goods that load positively on the global factor, global upturns improve export revenues, relax borrowing constraints, and lower spreads. If instead the country imports those goods, the same global upturn worsens its trade balance, increases borrowing costs, and makes default more likely. Hence, the response to global shocks depends critically on whether a country is an exporter or importer of goods that contribute strongly to the global factor.

The balance between these two forces, the price-index channel and the terms-of-trade income channel, defines an equilibrium *wedge* that governs how global shocks transmit to sovereign risk. When the terms-of-trade channel dominates, as in economies exporting globally priced goods, global upturns strengthen repayment incentives and compress spreads. When the price-index channel dominates, as in economies importing those same goods, the same upturns raise consumption costs, reduce real income, and widen spreads. This wedge thus provides a unified framework to interpret cross-country differences in the cyclicality of sovereign risk.

The model embeds these mechanisms within a standard dynamic default framework with risk-neutral lenders and endogenous borrowing limits. Countries differ only in their trade exposure and consumption structure, mapped from international price data and trade weights, so that the composition of trade alone determines which side of the wedge they fall on. Differences in leverage then amplify these effects, generating realistic heterogeneity in how exporters and importers respond to global price shocks.

Quantitatively, the model reproduces three key facts about sovereign risk and global cycles. First, positive global shocks lower default probabilities and compress spreads in exporter economies, while raising default probabilities and widening spreads for importers. Second, these patterns arise even when both economies share identical fundamentals, showing that trade composition alone can explain cross-country asymmetries in the cyclicality of spreads. Third, leverage endogenously amplifies the transmission: economies with smaller precautionary buffers display stronger sensitivity of default risk and borrowing costs to global factors. In simulations calibrated to international price data, the model generates realistic spread volatility and default frequencies, with exporter and importer economies responding with opposite signs to identical global shocks.

Together, these results imply that trade structure is a key determinant of sovereign resilience, and that policies enhancing export diversification or hedging against global price movements can mitigate default risk in import-dependent economies.

Related Literature. This paper connects three active literatures: (i) *the macroeconomic consequences of terms-of-trade and commodity-price shocks in small open economies*, (ii) *the quantitative sovereign default tradition with endogenous spreads*, and (iii) *modern shift–share identification strategies for isolating exposure to common shocks*.

Regarding *terms-of-trade and commodity-price shocks*, a large literature studies how external price movements shape macroeconomic dynamics, particularly in small open and commodity-exporting economies. In this literature, movements in export prices are typically interpreted through a nominal income channel: a fall in export prices reduces export revenues and welfare, whereas an increase boosts income and output. Classic contributions such as Mendoza (1995) show how terms-of-trade shocks generate output volatility and real-exchange-rate movements, with the exchange rate acting as a key adjustment mechanism. Subsequent work confirms that global price swings are a first-order driver of business-cycle fluctuations in emerging markets: Kose (2002) finds that world-price shocks explain nearly 90 percent of output variation across small open economies, while Caballero & Panageas (2008) and Calvo et al. (2008) link commodity-price declines to sudden stops in capital inflows and external crises. More recent quantitative studies such as Drechsel & Tenreyro (2018) show that commodity-price fluctuations can account for roughly one-third of output volatility in Argentina, through their effects on competitiveness and sovereign spreads.

A complementary line of research highlights the global nature of these price movements. Early work documented strong co-movement across markets (Pindyck & Rotemberg 1990), while factor analyses reveal pervasive "common forces" in primary-commodity prices (Byrne et al. 2013). In energy markets, Kilian (2009), Kilian & Murphy (2014), and Baumeister & Hamilton (2019) disentangle global demand and supply shocks, showing that a few persistent factors account for most commodity-price variation. This motivates our empirical and quantitative approach, in which global shocks are summarized by principal components of international prices.

We contribute to this literature by linking these global price factors to two structural transmission mechanisms: a price-index channel, which captures how global shocks change the cost of the consumption basket, and a terms-of-trade income channel, which reflects how trade composition shapes the income response to those shocks. Our key theoretical insight is a sign reversal: global upturns are expansionary for exporters but contractionary for importers.

On the *sovereign-risk* literature, our work builds on the Eaton–Gersovitz tradition of sovereign-default models (Eaton & Gersovitz 1981, Aguiar & Gopinath 2006, Arellano 2008). Important extensions have analyzed long-maturity bonds and pricing (Hatchondo & Martinez 2009, Chatterjee & Eyigungor 2012), renegotiation (Yue 2010), and the interaction between production, external adjustment, and default (Mendoza & Yue 2012). Empirically, global factors play a prominent role in determining sovereign spreads and CDS premia (González-Rozada & Yeyati 2008, Longstaff et al. 2011), consistent with the global financial-cycle literature (Rey 2015, Miranda-Agrippino & Rey 2020). Early studies had already emphasized the link between external prices and sovereign risk: Min (1998) and Cuadra & Sapriza (2006) show that terms-of-trade movements strongly affect default probabilities in emerging markets, while Hatchondo et al. (2007) document that the collapse in oil prices was central to Ecuador's 1999 default. These studies highlight the empirical relevance of commodity-price dynamics for debt sustainability, but they typically treat them as reduced-form income shocks.

Much of this literature focuses on export-driven economies, where higher commodity prices ease external financing conditions and lower risk premia. Empirical studies such as Hilscher & Nosbusch (2010) and Roch (2019) find that spreads are procyclical with commodity booms and busts, while models like Drechsel & Tenreyro (2018) replicate this mechanism for exporters. Yet recent evidence suggests that this view is incomplete: Di Pace et al. (2025) show that import- and export-price shocks have asymmetric macroeconomic effects, and that positive export shocks do not mirror negative import shocks.

We formalize this asymmetry within a sovereign-default framework, showing that for net importers, global price surges can raise risk premia even with identical fundamentals, because higher import costs erode real income and repayment capacity. Methodologically, our model replaces exogenous "income shocks" with observable global price factors and maps country-level exposures directly from trade weights and principal components of international prices. This allows us to decompose default incentives into the two structural channels above, explain why spread cyclicality flips between exporters and importers, and highlight an endogenous leverage term that scales transmission and generates heterogeneity in volatility and crisis frequency.

Beyond direct income effects, we connect to research on valuation and balance-sheet channels, which study how currency mismatches, original-sin constraints, and fiscal responses interact with sovereign risk (Eichengreen et al. 2005, Du & Schreger 2016, Catão & Milesi-Ferretti 2014). Our framework provides a structural interpretation of these mechanisms: the price-index channel captures cost-of-living pressures from rising import prices, whereas the terms-of-trade income channel captures changes in trade income from export prices.

On the *identification* literature, our empirical strategy contributes to the growing literature using *shift–share (Bartik-style) designs* to study heterogeneous exposure to common shocks. We interact global price innovations with lagged country-specific trade weights to capture import and export exposure. Recent methodological advances clarify identification and inference in such settings (Adao et al. 2019, Goldsmith-Pinkham et al. 2020, Borusyak et al. 2022). Similar designs have been applied to analyze the macroeconomic effects of commodity-price movements (e.g., Drechsel & Tenreyro 2018), but we extend this approach to sovereign spreads and external borrowing, explicitly distinguishing between import- and export-price shocks.

In summary, our contribution is twofold. Empirically, we provide causal evidence that the impact of global price shocks on sovereign risk depends on trade structure, distinguishing exporters from importers. Theoretically, we extend the standard sovereign-default framework to incorporate global price factors that enter both the consumption price index and trade income, allowing observed trade exposures to determine whether a given shock is expansionary or contractionary. This framework unifies diverse empirical findings on spread cyclicality and highlights how leverage and external buffers amplify asymmetries in sovereign vulnerability to global cycles.

The paper proceeds as follows. Section 2 presents the identification strategy, the data used and the results of the empirical analysis. In section 3, we present the theoretical framework, focusing on the two channels through which global commodity prices affect sovereign risk. In section 4, we discuss our estimation strategy and report the quantitative results. Section 5 concludes.

2 Empirical Evidence

2.1 Empirical Strategy

Our objective is to evaluate the effect of terms-of-trade movements on default risk and sovereign borrowing. We estimate regressions of the form:

$$\Delta y_{i,t} = \alpha_i + \beta_t + \gamma \Delta P_{i,t} + \delta X_{i,t} + \varepsilon_{i,t}, \tag{1}$$

where $\Delta y_{i,t}$ denotes the outcome of interest for country i in year t, for example, the change in sovereign spreads, and $\Delta P_{i,t}$ captures the change in the country's import or export prices.

Because countries in our sample are small relative to world markets, reverse causality is unlikely to bias the estimates: it is reasonable to view each country as a price taker. However, common global shocks could affect both commodity prices and sovereign spreads. For instance, a downturn in the global financial cycle may simultaneously depress commodity prices and raise spreads, inducing a negative bias in the OLS estimate of γ.

To address this concern, we implement a shift–share identification strategy that exploits cross-country heterogeneity in trade composition. The idea is analogous to an experiment in which global price shocks are randomly distributed across countries depending on their exposure to specific commodities. We define the shift–share instrument as:

$$z_{i,t} = \sum_{j=1}^{J} \omega_{i,j,t-1}, \varepsilon^P_{j,t}, \tag{2}$$

where $\omega_{i,j,t-1}$ is the share of commodity j in the total traded value (imports plus exports) of country i at time $t-1$.

The denominator includes both imports and exports, so the shares do not sum to one. As a result, the instrument is a weighted sum rather than a weighted average of global price shocks. Following Borusyak et al. (2022), we control for the total trade weight $\Omega_{i,t} = \sum_{j=1}^{J} \omega_{i,j,t-1}$ to avoid bias from correlation between $\Omega_{i,t}$ and the instrument. This construction allows us to exploit variation in the net trade position of each country. For example, if a country is highly specialized in a few export commodities and imports little, a positive price shock to its export basket will have a stronger effect on its terms of trade than on its import prices—an effect that would be muted if import and export denominators were treated separately.

The shifts, $\varepsilon^P_{j,t}$, represent the innovations in the log change of the price of commodity j between $t-1$ and t. We obtain these by modeling each commodity's log price change as an AR(k_j) process and taking the residuals:

$$\Phi_{k_j}(L)\Delta p_{j,t} = \varepsilon^P_{j,t}. \tag{3}$$

This procedure ensures that the shocks represent the unanticipated component of global price changes. Intuitively, the "surprise" component of price variation between $t-1$ and t could not have influenced the lagged trade shares $\omega_{i,j,t-1}$, satisfying the exclusion restriction. Moreover, because predictable components of price changes are removed up to $t-1$, using one lag of trade shares suffices.

Overall, the instrument captures exogenous cross-country variation in exposure to global price shocks. Countries differ in their trade composition, and these differences translate into heterogeneous exposure to unanticipated movements in world prices.

Finally, we include a baseline set of controls to account for remaining macroeconomic channels that may correlate with commodity prices. Specifically, we control for real GDP growth, CPI inflation, and the USD–domestic currency exchange rate. These variables absorb global or domestic factors—such as inflation pass-through from import prices—that could otherwise bias the estimated effect of price shocks on sovereign spreads.

2.2 Data description

The analysis covers 37 emerging and developing economies over the period 1991–2023.[1] We construct country-specific terms-of-trade indices based on 41 internationally traded commodities — ranging from energy products (crude oil, natural gas, coal) and metals (copper, aluminum, iron ore, zinc) to agricultural goods (wheat, rice, coffee, cocoa, soybeans) and livestock products (beef, pork, chicken, wool). Commodity price data are taken from the *IMF Primary Commodity Prices Database*, while trade weights are derived from bilateral trade flows reported in *UN Comtrade*. Macroeconomic and financial variables are drawn from various IMF sources: real and nominal GDP from the *World Economic Outlook*, external balance and debt indicators from the *International Financial Statistics (IFS)* and the *International Debt Statistics (IDS)*, and sovereign spreads (EMBI) from the *Global Financial Database (GFD)* via Fineanon access.

2.3 Shift-Share Instrument

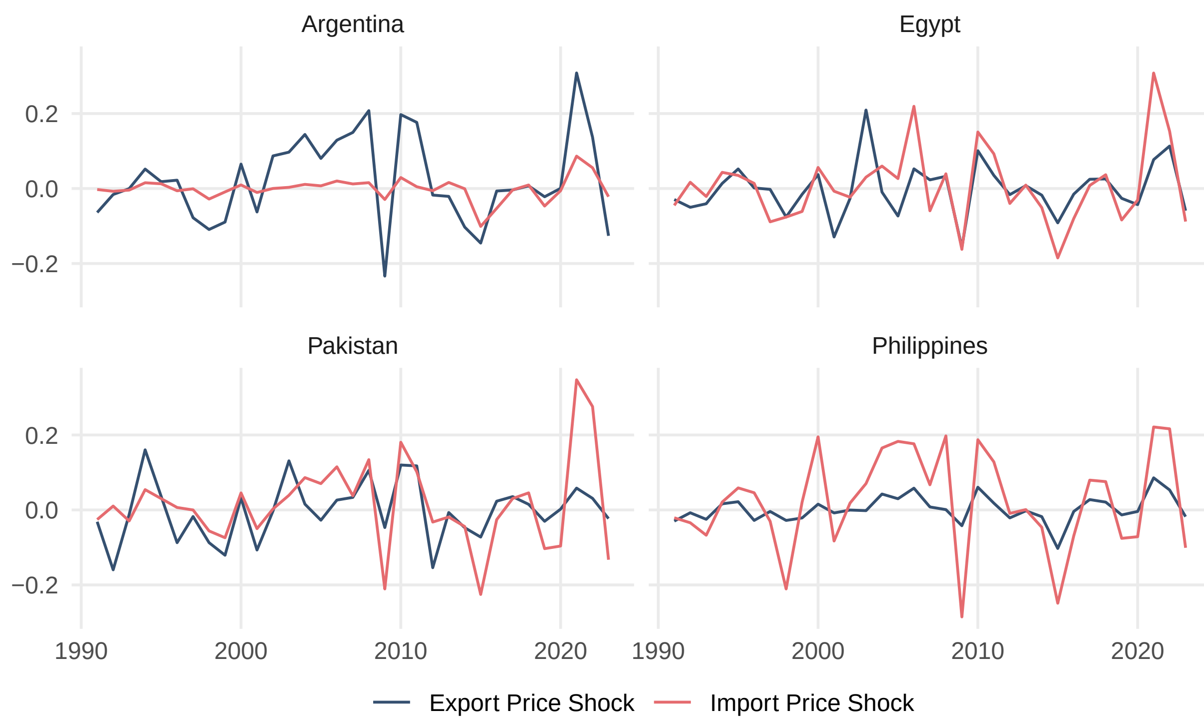
Figure 1 illustrates the shift–share instruments for import and export price shocks across countries. The identifying variation arises from heterogeneous trade exposure to movements in global commodity prices. In economies that primarily export commodities, such as Argentina, fluctuations in world prices translate into strong export-price shocks. Conversely, in

[1] The dataset covers: Argentina, Belize, Brazil, Bulgaria, Chile, China, Colombia, Dominican Republic, Ecuador, Egypt, El Salvador, Gabon, Ghana, Hungary, Indonesia, Iraq, Jamaica, Kazakhstan, Lebanon, Malaysia, Mexico, Pakistan, Panama, Peru, Philippines, Poland, Russia, Serbia, South Africa, South Korea, Sri Lanka, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam.

import-dependent economies such as the Philippines, the same global swings mainly identify import-price shocks.

Because commodity prices co-move strongly, part of the variation is absorbed by time fixed effects. For example, the common collapse in commodity prices in 2009 generates a large negative shock for both exporters and importers. Nonetheless, the idiosyncratic component of each commodity price, combined with cross-country differences in trade composition, still provides rich identifying variation across countries and over time. An illustrative case is 1994, when the price of cotton surged: Pakistan, where cotton exports accounted for nearly half of the total value of traded commodities in our sample, experienced a pronounced positive export-price shock, while Argentina, Egypt, and the Philippines, where cotton exports were negligible, were largely unaffected.

Figure 1: Shift-Share Instrument.



Note: This figure shows our shift-share measure of the import and export price shocks.

2.4 Results

We now present the baseline results on the effects of import and export price shocks on sovereign risk. Table 1 reports the estimates for changes in sovereign spreads.

Columns (1) and (2) show the results when import and export price shocks are included separately. A one–standard-deviation increase in import prices raises sovereign spreads by 77.7 basis points, a statistically significant effect. In contrast, export price shocks have a smaller and noisier effect of –47 basis points.

When both shocks are included jointly (column 3), the effect of import prices remains large and significant (68.9 basis points), while the export-price coefficient shrinks to –11.9 basis points and becomes insignificant. This result is robust to the inclusion of additional control variables (column 4).

To ensure that extreme crises are not driving the results, column (5) restricts the sample to changes in spreads below 500 basis points. The estimated effect of import prices remains positive and significant at 28 basis points, whereas the export-price coefficient is close to zero (1.7 basis points) and statistically insignificant.

Finally, column (6) limits the sample to countries with external-debt-to-output ratios below 30%. The impact of import price shocks declines modestly to 57.2 basis points but remains highly significant, while the export-price effect (5.5 basis points) remains negligible and statistically insignificant.

Table 1: Baseline Results EMBI Spreads.

	Δ spread					
	(1)	(2)	(3)	(4)	(5)	(6)
Import Price Shock	77.7		68.9	70.9	28.0	57.2
	(28.3)		(22.6)	(28.9)	(13.2)	(29.6)
Export Price Shock		-47.0	-11.9	-20.6	1.66	5.47
		(27.9)	(25.3)	(33.8)	(9.81)	(8.02)
Observations	511	517	511	375	484	306
R^2	0.35	0.35	0.35	0.41	0.51	0.48
Country fixed effects	✓	✓	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓	✓	✓
Extra controls				✓		
Low spreads					✓	
Low debt						✓

Note: This table shows the estimates of import and export price shocks, measured by a shift-share instrument, on the change in EMBI spread. All regressions control for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. The extra controls include the growth rate of the current account, the average maturity, and the growth rate in interest and principal debt repayments. The 'low spreads' sample restricts changes in spreads to be smaller than 500bp. The 'low debt' sample restricts external debt to output ratio to be smaller than 30%. The shocks are normalized to one standard deviation. Standard errors are clustered at the country level.

We next examine the effect of commodity price shocks on the flow of external debt, defined as new borrowing by the general government. The nominal flow of debt is normalized by nominal GDP. Table 2 reports the estimates.

Across all specifications, both import and export price shocks are associated with a decline in new external debt. However, these effects are imprecisely estimated and not statistically significant. When the sample is restricted to countries with external-debt-to-output ratios below 30% (column 6), the estimated effects of both shocks are close to zero. For import price shocks, this result is consistent with the earlier evidence on spreads: higher borrowing costs tighten financing conditions and limit new debt issuance.

Table 2: Baseline Results on New External Debt.

	Δ debt					
	(1)	(2)	(3)	(4)	(5)	(6)
Import Price Shock	-0.247		-0.585	-0.385	-0.030	-0.016
	(0.185)		(0.347)	(0.179)	(0.142)	(0.194)
Export Price Shock		-0.107	-0.423	-0.282	-0.161	-0.021
		(0.173)	(0.299)	(0.143)	(0.105)	(0.104)
Observations	755	757	755	753	351	520
R^2	0.37	0.37	0.38	0.62	0.57	0.42
Country fixed effects	✓	✓	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓	✓	✓
Extra controls				✓		
Low spreads					✓	
Low debt						✓

Note: This table shows the estimates of import and export price shocks, measured by a shift-share instrument, on the new external debt received by the general government. All regressions control for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. The extra controls include the growth rate of the current account, the average maturity, and the growth rate in interest and principal debt repayments. The 'low spreads' sample restricts changes in spreads to be smaller than 500bp. The 'low debt' sample restricts external debt to output ratio to be smaller than 30%. The shocks are normalized to one standard deviation. Standard errors are clustered at the country level.

2.5 Robustness Checks

As a first robustness check, we construct the shift–share instrument excluding energy commodities (crude oil, natural gas, and coal). In the case of exporters, identification could be threatened if a country in the sample is large enough to influence global energy prices.

More generally, it is informative to assess whether the baseline results are primarily driven by energy commodities.

Table 3 reports the estimates for the effect on sovereign spreads. When considering import price shocks alone (column 1), the effect declines to 19.1 basis points and becomes statistically insignificant. However, when both import and export shocks are included jointly (column 3), the effect of import price shocks remains close to the baseline at 53.8 basis points and is statistically significant. The coefficient on export price shocks is positive in all specifications and statistically significant when both shocks are included (41.6 basis points, column 3).

Table 3: Effect on Spreads, Excluding Energy Commodities.

	Δ spread					
	(1)	(2)	(3)	(4)	(5)	(6)
Import Price Shock	19.1		53.8	53.7	23.2	61.2
	(19.9)		(19.9)	(25.0)	(9.31)	(25.0)
Export Price Shock		21.0	41.6	26.6	19.5	37.1
		(15.2)	(16.9)	(24.3)	(9.84)	(19.6)
Observations	511	515	509	375	482	306
R^2	0.34	0.34	0.35	0.40	0.52	0.49
Country fixed effects	✓	✓	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓	✓	✓
Extra controls				✓		
Low spreads					✓	
Low debt						✓

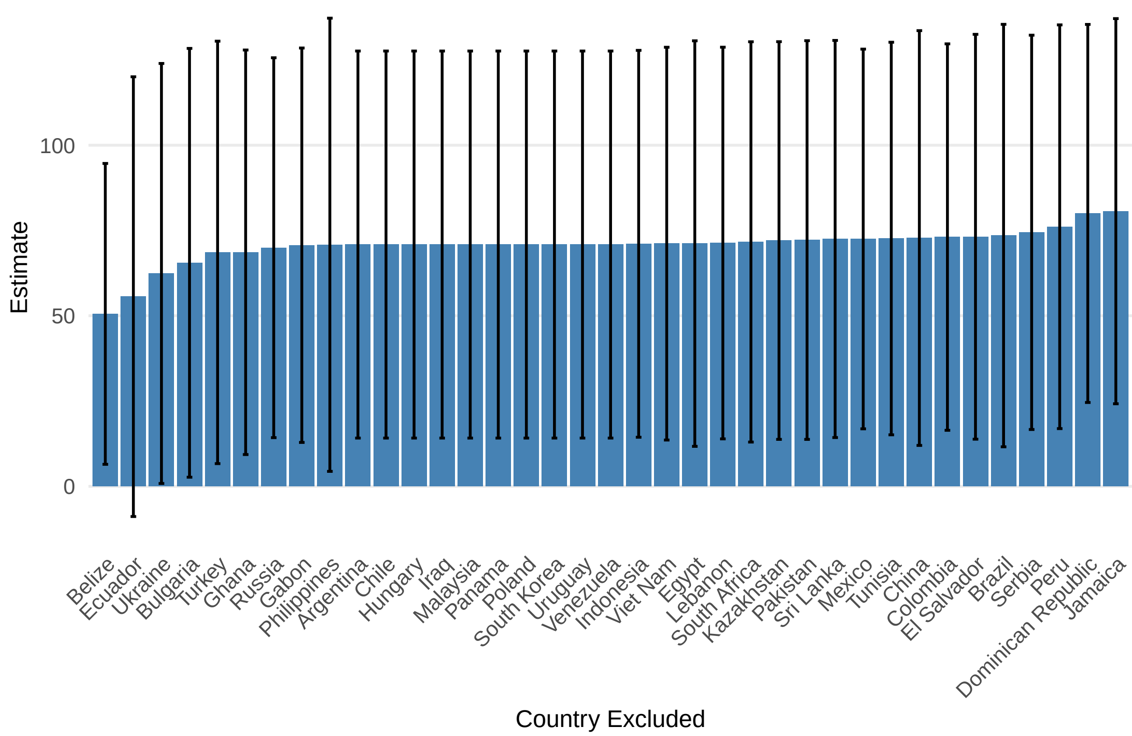
Note: This table shows the estimates of import and export price shocks, measured by a shift-share instrument, on the change in EMBI spread. All regressions control for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. The extra controls include the growth rate of the current account, the average maturity, and the growth rate in interest and principal debt repayments. The 'low spreads' sample restricts changes in spreads to be smaller than 500bp. The 'low debt' sample restricts external debt to output ratio to be smaller than 30%. The shocks are normalized to one standard deviation. Standard errors are clustered at the country level. The shift-share instrument is constructed excluding energy commodities.

In Appendix A, Table 6 reports the corresponding estimates for new external debt. The results are similar to the baseline: both import and export price shocks are associated with a decline in new external debt, but the estimates are imprecise and statistically insignificant.

We next assess whether the results are driven by any single country in the sample. Figure 2 reports the point estimates and 95% confidence intervals for the effect of import price shocks on sovereign spreads, excluding one country at a time from the estimation sample. The specification corresponds to column 3 in Table 1, which includes both import and export price shocks as well as the full set of controls.

The countries exerting the largest influence on the estimates are Belize, Ukraine, the Dominican Republic, and Jamaica. Overall, the results are stable across exclusions. Excluding Belize reduces the estimated effect of import price shocks from the baseline value to about 50 basis points, while exclusions of other countries have negligible effects. Estimates for export price shocks display similar stability when excluding one country at a time, as shown in Figure 7 in Appendix A.

Figure 2: Leave-One-Out Estimates: Import Price Shocks.



Note: This figure shows the effect of import price shocks and sovereign spreads, excluding one country at a time from the sample. Each vertical bar corresponds to the country excluded. The horizontal brackets show the 95% confidence interval. The specification includes both import and export price shocks, and controls for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. Standard errors are clustered at the country level.

3 Model

Time is discrete and the horizon is infinite. We study a small open economy in which the government chooses borrowing and consumption while facing global price movements that shift both the cost of the consumption basket and the value of net exports. International lenders are risk-neutral and price sovereign bonds competitively based on default risk; default triggers temporary market exclusion and a deterioration in the country's terms of trade.

Relative to Arellano (2008), the key innovation lies in the nature and transmission of shocks. Instead of exogenous income fluctuations, the economy is subject to *global price shocks* that affect both domestic absorption and the relative price of traded goods. These shocks operate through two structural mechanisms: a *price-index channel*, which alters the cost of the consumption basket, and a *terms-of-trade channel*, which changes national income through trade exposure. Depending on its trade structure, a positive global shock can be expansionary, easing borrowing conditions for an exporter, or contractionary, tightening credit for an importer.

3.1 Environment

Goods, Prices, and Global Price Shocks. Consider a small open economy with a continuum of tradable goods indexed by $i \in [0,1]$. Each good i has a price $P_t(i)$ and a subsistence level $\bar{c}(i)$. Movements in world commodity markets are summarized by a set of K *global factors* capturing broad co-movements in international prices:

$$\ln P_t(i) = \mu(i) + a(i)^\top f_t + \varepsilon_t(i), \tag{4}$$

where $\mu(i)$ is the time-invariant mean of $\ln P_t(i)$; f_t collects the first K principal components of global commodity prices; $a(i)$ are the associated factor loadings capturing how each good co-moves with the *global factors*; and $\varepsilon_t(i)$ is an idiosyncratic residual. The factors follow a stationary VAR(1):

$$f_t = \Phi f_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \Sigma_u). \tag{5}$$

Preferences and Aggregation. Households consume all goods according to a Stone–Geary aggregator:

$$c_t = \prod_{i\in[0,1]} [c_t(i) - \bar{c}(i)]^{\lambda(i)}, \qquad \int_0^1 \lambda(i)\, di = 1,$$

where $\bar{c}(i)$ denotes subsistence levels and $\lambda(i)$ the relative weight of each good in the consumption basket.

Budget Constraint and Trade. Let $e(i)$ denote endowments and D_t the net foreign asset position. Aggregate expenditure satisfies:

$$E_t = \int_0^1 P_t(i)e(i)\, di + D_t = \int_0^1 P_t(i)c_t(i)\, di.$$

Define the net endowment profile $\tilde{e}(i) \equiv e(i) - \bar{c}(i)$, yielding the resource constraint:

$$E_t - \int_0^1 P_t(i)\bar{c}(i)\,di = D_t + \int_0^1 P_t(i)\tilde{e}(i)\,di. \tag{6}$$

Aggregate Consumption. Aggregating across goods yields:

$$c_t = \Gamma^c\, e^{-\mu^C - \psi^\top f_t}\Big(D_t + \bar{Z} + \beta^\top f_t\Big), \tag{7}$$

where $\Gamma^c \equiv \prod_i \lambda(i)^{\lambda(i)}$ normalizes the consumption index; $\mu^C \equiv \int_0^1 \lambda(i)\mu(i)\,di$ is the average log price level; $\psi \equiv \int_0^1 \lambda(i)a(i)\,di$ summarizes the basket's exposure to global prices (the *price-index exposure*); $\bar{Z} \equiv \int_0^1 \bar{P}(i)\tilde{e}(i)\,di$ measures baseline trade income evaluated at steady-state prices $\bar{P}(i) \equiv e^{\mu(i)}$; and $\beta_k \equiv \int_0^1 \bar{P}(i)\tilde{e}(i)a_k(i)\,di$ captures how the country's trade composition aligns with global factors (the *terms-of-trade exposure*). A detailed derivation of this expression, including the price index, demand system, and intermediary steps, is provided in Appendix B.

Global shocks affect consumption through two channels: (i) a *price-index effect* $(-\psi^\top f_t)$, which raises the cost of consumption; and (ii) a *terms-of-trade effect* $(+\beta^\top f_t)$, which shifts income via net exports. Holding external resources fixed, the elasticity of consumption with respect to factor $f_{k,t}$ is:

$$\frac{\partial \widehat{\ln c_t}}{\partial f_{k,t}} = -\psi_k + \underbrace{\frac{1}{\bar{D} + \bar{Z}}}_{\kappa_c}\,\beta_k.$$

Consumption increases when the income effect dominates the price effect, that is, when $\beta_k > (\bar{D} + \bar{Z})\psi_k$. The factor $\kappa_c = 1/(\bar{D} + \bar{Z})$ acts as an endogenous leverage term: economies with smaller external buffers respond more strongly to global shocks.

Intuitively, when external resources are constrained, any improvement in export prices or deterioration in import costs translates into larger fluctuations in real consumption and default risk. In equilibrium, these forces shape the government's incentives to default. When the country is a net exporter of high-loading goods, favorable global shocks raise consumption, reduce default risk, and compress sovereign spreads. Conversely, for import-dependent economies, the same shocks can be stagflationary, raising the consumption price index, reducing real income, and increasing default risk. These mechanisms are developed in the next section.

3.2 Government Problem and Market Structure

A benevolent government maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{8}$$

where expectations are taken with respect to the realizations of the global shocks, $0 < \beta < 1$ is the discount factor, and $u(c)$ is a strictly increasing and concave period utility function. Aggregate consumption c_t is given by equation (7). The government's optimization is subject to the aggregate resource constraint and the bond-market equilibrium conditions.

Timing, States, and Resource Flows. At the beginning of period t, the state of the economy is (f_t, b_t), where f_t denotes the realization of the global factor and b_t the stock of sovereign debt carried from the previous period. After observing f_t, the government decides whether to *repay* or *default*. If it repays, it retains market access and chooses next-period debt b_{t+1}. Positive values of b_t correspond to net foreign assets, while negative values indicate outstanding debt.

Resource flows between the country and the rest of the world are:

$$D_t = b_t - q(f_t, b_{t+1})\, b_{t+1}, \tag{9}$$

with $D_t > 0$ indicating net borrowing. Risk-neutral international lenders price bonds competitively to satisfy zero expected profits:

$$q(f_t, b_{t+1}) = \frac{1 - \delta(f_t, b_{t+1})}{1 + r^*}, \tag{10}$$

where $\delta(f_t, b_{t+1})$ is the endogenous probability of default. The implied sovereign spread is $s_t = 1/q(f_t, b_{t+1}) - (1 + r^*)$. In default, the economy enters financial autarky and re-enters credit markets with probability $\theta \in (0, 1)$.

3.3 Recursive Competitive Equilibrium

Given the state vector $s = (f, b)$, the government's policy $b'(f, b)$, the bond price $q(f, b')$, and the implied consumption $c(f, b)$, we define the equilibrium.

Value Functions. Let $V^r(f, b)$ and $V^d(f)$ denote the value functions under repayment and default, respectively:

$$V(f, b) = \max\{V^r(f, b), V^d(f)\}. \tag{11}$$

Under repayment, the government retains market access and optimally chooses b':

$$V^r(f,b) = \max_{b'} \left\{ u(c) + \beta \int V(f',b')\, f(f' \mid f)\, df' \right\}, \tag{12}$$

$$c = \Gamma^c \exp\Big(-\mu^C - \psi^\top f\Big) \Big[b - q(f,b')\, b' + \bar{Z} + \beta^\top f \Big], \tag{13}$$

subject to a borrowing limit $b' \geq -\underline{b}$, which rules out Ponzi schemes and does not bind in equilibrium.

If the government defaults, it loses access to credit and faces costlier imports through higher effective prices for consumption goods. While in autarky, it cannot borrow or lend but regains access with probability $\theta \in (0,1)$. The corresponding value is:

$$V^d(f) = u(c^d) + \beta \int \Big[\theta\, V(f',0) + (1-\theta)\, V^d(f') \Big] f(f' \mid f)\, df', \tag{14}$$

$$c^d = \chi\, \Gamma^c \exp\Big(-\mu^C - \psi^\top f\Big) \Big(\bar{Z} + \beta^\top f \Big), \tag{15}$$

where $\chi \in (0,1)$ captures the consumption loss during default.

Default and Repayment Sets. Define the default and repayment regions as:

$$A(b) = \{ f \in \mathbb{F} : V^r(f,b) \geq V^d(f) \}, \qquad D(b) = \{ f \in \mathbb{F} : V^r(f,b) < V^d(f) \},$$

where $\mathbb{F}$ is the support of f. The probability of default implied by these regions is:

$$\delta(f,b') = \int_{D(b')} f(f' \mid f)\, df'. \tag{16}$$

When $b' \geq 0$, default is impossible ($\delta = 0$); when $D(b') = \mathbb{F}$, default is certain.

Definition 1. *A* ***recursive competitive equilibrium*** *consists of functions for (i) consumption $c(f,b)$ and $c^d(f)$, (ii) debt policy $b'(f,b)$ and repayment and default sets $A(b)$ and $D(b)$, and (iii) bond prices $q(f,b')$, such that:*

1. *Given policies, $c(f,b)$ and $c^d(f)$ satisfy the intertemporal resource constraint.*
2. *Given $q(f,b')$, the policy $b'(f,b)$ and sets $A(b)$ and $D(b)$ solve the government problem in (11)–(15).*
3. *Bond prices satisfy the zero-profit condition (10).*

Discussion. The equilibrium jointly determines debt, default, and consumption as functions of the global state f. Global shocks affect sovereign spreads and default risk through the same mechanisms described earlier: a *price-index channel* (via ψ) and a *terms-of-trade* *income channel* (via β). Export-oriented economies benefit from favorable global shocks, while import-dependent ones face higher prices and tighter credit conditions. We now formalize how global shocks map into equilibrium consumption and default risk.

Proposition 1 (Global factors, consumption, and default incentives)**.** *Let $f \in \mathbb{R}^K$ denote the vector of global price factors driving the economy, and let aggregate consumption in equilibrium be:*

$$c(f,b) = \Gamma^c \exp(-\mu^C - \psi^\top f)\left(D(f,b) + \bar{Z} + \beta^\top f\right),$$

where $D(f,b)$ is the equilibrium net resource flow consistent with policy and bond pricing. Define the consumption wedge*:*

$$\Delta_k(f,b) \equiv \frac{\partial \ln c(f,b)}{\partial f_k} = -\psi_k + \kappa_c(f,b)\,\beta_k, \qquad \kappa_c(f,b) \equiv \frac{1}{D(f,b)+\bar{Z}}.$$

Then:

(i) ***Sign.*** *A marginal increase in factor f_k raises equilibrium consumption if $\Delta_k(f,b) > 0$ and lowers it if $\Delta_k(f,b) < 0$. The term $-\psi_k$ captures the* price-index channel*, while $\kappa_c(f,b)\beta_k$ captures the* terms-of-trade income channel*.*

(ii) ***Amplification.*** *The leverage coefficient $\kappa_c(f,b)$ amplifies the income channel; smaller buffers imply stronger sensitivity.*

(iii) ***Default incentives.*** *If $\Delta_k(f,b) > 0$, favorable shocks improve repayment incentives and compress spreads; if $\Delta_k(f,b) < 0$, they weaken repayment incentives and raise default risk.*

Sketch of proof. Differentiate $\ln c(f,b)$ with respect to f_k using the equilibrium resource constraint. The derivative decomposes into a price-index effect ($-\psi_k$) and a trade-income effect ($\kappa_c(f,b)\beta_k$). Since V^r is strictly increasing in c, the sign of $\Delta_k(f,b)$ determines the impact on default incentives. A detailed proof is provided in Appendix C.1. □

Remark 1 (Exporter vs. importer exposure)**.** *When $\beta_k > 0$, the country is a net* exporter *in goods with high loadings on factor k: global upturns raise income and reduce default risk. When $\beta_k < 0$, it is a net* importer*: similar shocks worsen the terms of trade and raise default risk. The relative magnitudes of ψ_k and $\kappa_c(f,b)\beta_k$ determine whether the shock is expansionary or contractionary.*

Extensions linking the consumption wedge $\Delta_k(f,b)$ to sovereign spreads (Corollary 1) and to global comparative statics under bounded leverage (Lemma 1), which establish how the qualitative results extend to large and persistent global shocks, are provided in Appendix C.2 and Appendix C.3, respectively. Lemma 1 shows that when either the *price-index* or the *terms-of-trade* channel dominates uniformly, the qualitative effects of global shocks extend beyond local approximations.

This result provides a robust bridge between the theoretical mechanism and the quantitative analysis that follows. In the calibrated model, countries differ primarily in their exposure parameter β, and the sign of Δ_k determines how large, persistent movements in global prices translate into macroeconomic and financial outcomes.

4 Quantitative Analysis

4.1 Calibration and Estimation Strategy

Overview. The model is calibrated at a quarterly frequency to match key features of the data on global prices and sovereign borrowing. The calibration draws directly on the empirical strategy presented in Section 2, which identifies the principal components of international commodity prices and the country's exposure to global shocks.

Preferences and International Finance. Households (or the representative government) are assumed to have CRRA preferences:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{17}$$

with risk aversion parameter $\sigma = 2$. The subjective discount factor is $\beta = 0.97$. The international risk-free rate is fixed at $r^* = 1\%$, and foreign creditors are risk-neutral. The probability of regaining market access after a default is $\theta = 0.1$, in line with typical values for emerging economies, and the default penalty parameter is $\chi = 0.9$, reducing effective consumption during exclusion. To prevent Ponzi schemes, we impose a lower bound on sovereign debt, $b' \geq -\underline{b}$, where $\underline{b}$ = -0.9, assumed non-binding in equilibrium.

Global Price Factors. Following the empirical procedure described in Section 2, we use a principal component analysis (PCA) on the panel of global log commodity prices $\ln P_t(i)$ to extract the common factors that capture worldwide fluctuations in tradable goods prices. Specifically, the log price of each good is represented as:

$$\ln P_t(i) = \mu(i) + a(i)^\top f_t + \varepsilon_t(i), \tag{18}$$

where $\mu(i)$ is the average log price of good i, $a(i)$ the vector of factor loadings, and $\varepsilon_t(i)$ an idiosyncratic component.

The empirical analysis shows that the first principal component explains about 50% of the total variance in international prices, while the first seven components together account for approximately 90% (see Figure 8 in Appendix A). Given the high explanatory power of the first component, we set the number of global factors in the model to $K = 1$. This choice captures the dominant common source of variation in international prices, interpreted as a global business-cycle or terms-of-trade factor.

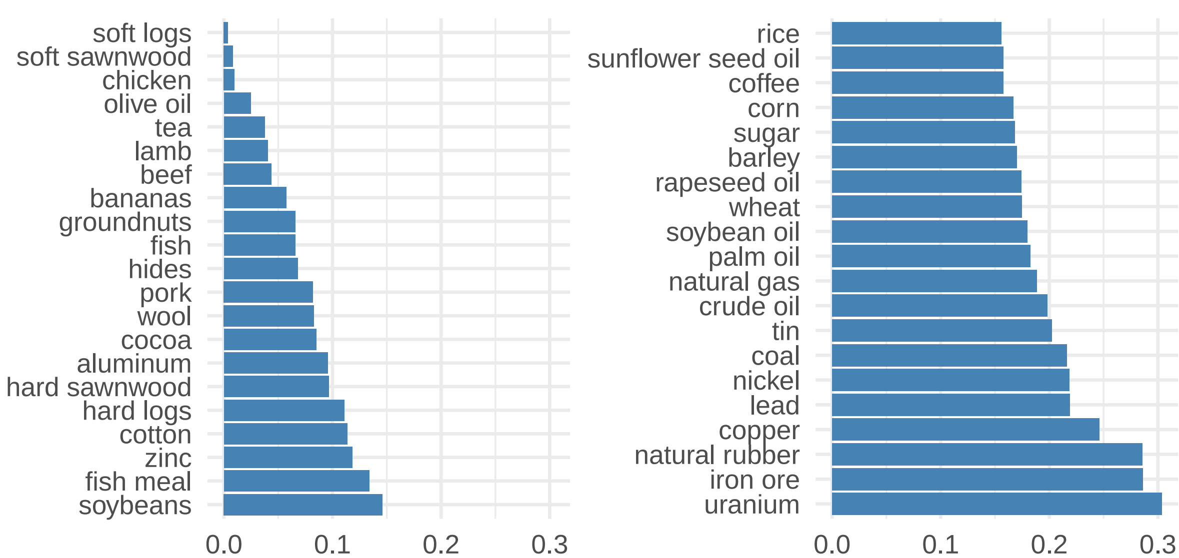
The factor follows a stationary autoregressive process:

$$f_t = \rho_f f_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \sigma_f^2), \tag{19}$$

where $\rho_f = 0.97$ denotes persistence and $\sigma_f = 0.24$ the standard deviation of shocks. The process is estimated from the time series of the first principal component and discretized using the quadrature method of Tauchen & Hussey (1991). Details of the estimation procedure are provided in Appendix D.

Figure 3 presents the loadings of each commodity on the first principal component. The exposure of commodities to the global factor is highly heterogeneous. Some commodities, such as energy products and minerals, are strongly influenced by global shocks, whereas others, such as many agricultural goods, are much less sensitive. We exploit this heterogeneity in our quantitative analysis to simulate different trade structures that capture varying degrees of exposure to the global shock.

Figure 3: Factor Loadings.



Note: This figure shows the loadings of each commodity price on the first principal component.

Trade Structure and Exposure Parameters. The model requires two key exposure parameters: the consumption-price index exposure ψ and the net-export exposure β. These are computed as:

$$\psi = \int_0^1 \lambda(i)a(i)\,di, \qquad \beta = \int_0^1 \bar{P}(i)\tilde{e}(i)a(i)\,di, \tag{20}$$

where $\lambda(i)$ denotes the consumption weights, $\tilde{e}(i) = e(i) - \bar{c}(i)$ the net-export profile of good i, and $\bar{P}(i) = \exp(\mu(i))$ its baseline price. We construct two distinct trade structure profiles for our experiments, an exposed exporter country and exposed importer country. By exposed, we mean that a country has a large trade position (whether exporter or importer) in commodities with large loadings, which are more exposed to the global factor. We focus on the net endowment $\tilde{e}(i)$ as a source of variation in trade exposure. That is, we assume equal consumption weights $\lambda(i)$ across commodities, so that $\psi = 0.13$ in both scenarios. For the exporter case, we assume that the net endowment is such that $\beta = 0.1$. This implies that the country has a large endowment in commodities more exposed to the global factor, and hence will export more of them. In the importer case, we assume that the net endowment is such that $\beta = -0.1$, meaning that the country imports more of the commodities more exposed to the global factor.

The steady-state trade term $\bar{Z}$ is given by:

$$\bar{Z} = \int_0^1 \bar{P}(i)\tilde{e}(i)\,di, \tag{21}$$

and the steady-state consumption price index constant is:

$$\mu^C = \int_0^1 \lambda(i)\mu(i)\,di. \tag{22}$$

In the quantitative implementation, these parameters are computed empirically from $N = 40$ commodities using the estimated factor loadings $a(i)$ together with simulated trade and consumption weights. The resulting values of ψ and β for each country are reported in Table 4. This approach ensures consistency between the empirical estimation of exposures and the model calibration. Details of the procedure are provided in Appendix D.

Parameter Summary. All parameter values used in the quantitative exercises are summarized in Table 4.

Table 4: Model Parameters and Exposure Estimates.

Parameter	**Symbol / Definition**	**Value**
Risk aversion	σ	2
Discount factor	β	0.97
Risk-free rate	r^*	1%
Persistence of global factor	ρ_f	0.97
Volatility of global factor	σ_f	0.24
Re-entry probability	θ	0.1
Default penalty	χ	0.9
Steady-state trade term	$\bar{Z}$	1
Debt bounds	$b \in$	$[-0.9, 0.9]$
Exposure parameters:		
Consumption exposure	ψ	0.13
Net-export exposure (Exporter)	β^A	0.1
Net-export exposure (Importer)	β^B	-0.1

Note: The debt grid b consists of 400 equally spaced points, and the global factor process f_t is discretized into 60 nodes using the quadrature method of Tauchen & Hussey (1991).

Numerical Implementation. Given $(\psi, \beta, \bar{Z}, \mu^C)$, we solve for the recursive equilibrium through value function iteration and the lenders' zero-profit condition. Appendix D details the estimation of exposure parameters and the full numerical procedure.

4.2 Quantitative Results

Overview. Using the calibration from Section 4.1, we compare two economies that differ only in their trade exposure: (i) a net **exporter** of goods with high positive loadings on the global factor ($\beta > 0$), and (ii) a net **importer** of such goods ($\beta < 0$). Both economies are otherwise identical.

The analysis illustrates how global shocks affect sovereign borrowing, default risk, and welfare under these contrasting profiles, consistent with Proposition 1 and Lemma 1.

We proceed in two steps. First, we describe default decisions and the corresponding bond price schedules in equilibrium. Second, we analyze the dynamic propagation of global shocks through simulations of the calibrated model and compare the resulting moments across the two economies, focusing on sovereign spreads, debt levels, and consumption.

4.2.1 Default Incentives and Bond Prices

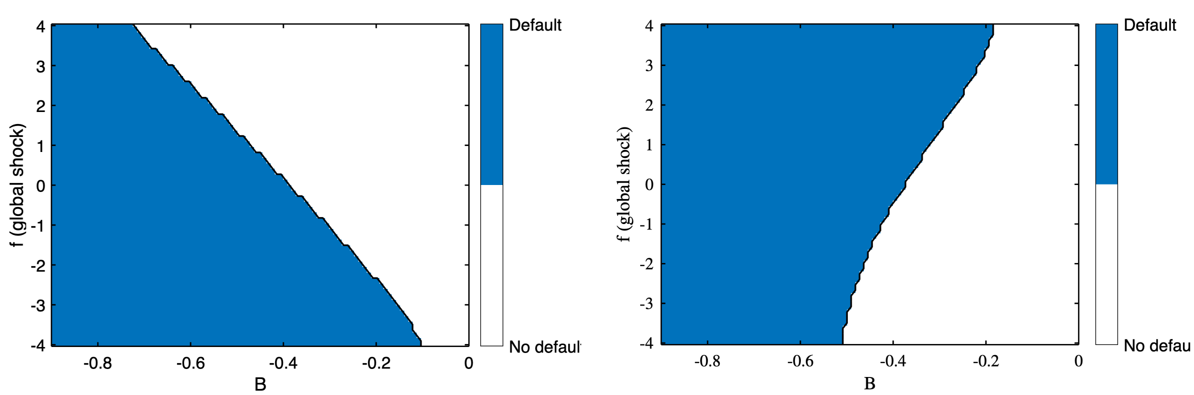
Figure 4 presents the default sets in both economies. In both cases, higher levels of indebtedness increase default incentives, as more resources are freed for consumption upon default. However, the effect of the global factor differs across the two economies.

In the exporter economy, an increase in the global factor reduces default incentives. This economy is characterized by large positive trade positions in commodities that are highly exposed to the global factor. As a result, expansions in the global commodity cycle improve the terms of trade. In the calibration, this positive income effect dominates the adverse cost-of-living channel, leading to higher real consumption when the global factor rises. Because default is optimal when the marginal utility of consumption is high (that is, when consumption is low), higher global factors reduce the government's incentive to default. Hence, in the exporter case, default incentives are strongest when the global factor is low, as falling export prices depress real income and consumption.

In the importer economy, by contrast, default incentives increase with the global factor. Since the country imports the commodities most exposed to the global factor, a global boom worsens its terms of trade and raises the domestic price index. Both channels reduce real consumption and increase the marginal utility of consumption in states where the global factor is high, thereby heightening default risk.

Finally, default incentives in the exporter economy are more sensitive to the global factor than in the importer economy. As shown in Figure 4, the default frontier is a steeper function of the global factor in the exporter case.

Figure 4: Default Sets.

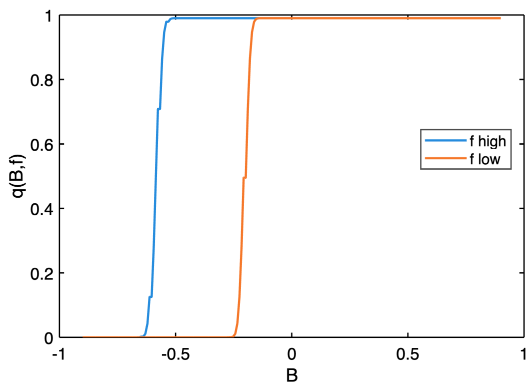


(a) Exporter Case.

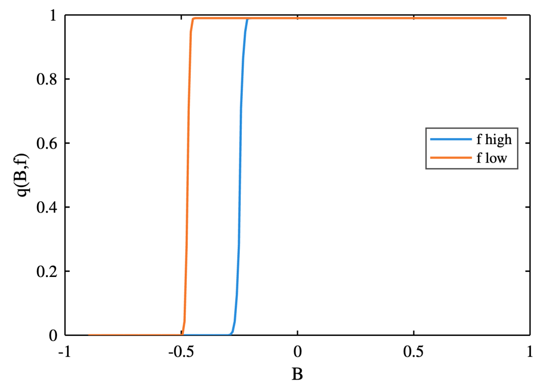
(b) Importer Case.

In equilibrium, default incentives are reflected in the bond price schedule, shown in Figure 5. The figure illustrates the opposite effects of the global factor on bond prices and, consequently, on sovereign spreads. In the exporter economy, an increase in the global factor raises bond prices and therefore reduces spreads. By contrast, in the importer economy, higher global factors lower bond prices and increase spreads.

Figure 5: Bond Price Schedules.



(a) Exporter Case.



(b) Importer Case.

4.2.2 Dynamic Transmission of Global Shocks

We now examine the dynamic properties of the model and the propagation of global shocks. We simulate 500 economies for both the exporter and importer cases, each for 10,000 periods. Table 5 reports the main simulated moments for the two economies.

Under both calibrations, the model generates high levels of sovereign debt, around 0.32 relative to an average consumption normalized to 1. Such levels can be sustained in equilibrium because default entails a direct consumption cost and because the large volatility of the global factor strengthens the sovereign's motive to smooth consumption, making autarky more costly.

Table 5: Simulation Moments Across Two Calibrations.

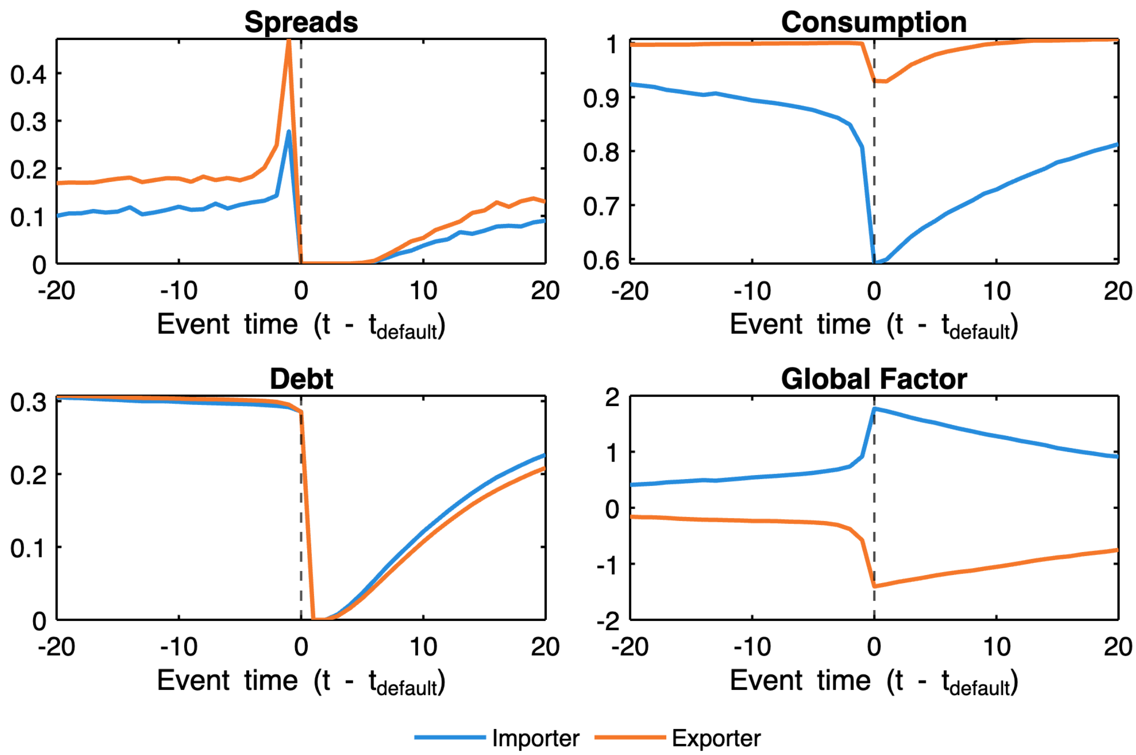
	Importer $\beta < 0$			Exporter $\beta > 0$		
Variable	Mean	Std	Cov w/ F	Mean	Std	Cov w/ F
Annual Spread (%)	0.07	0.19	0.07	0.16	0.24	-0.1
Debt	0.32	0.05	0.04	0.32	0.08	0.07
Consumption	1.02	0.25	-0.25	0.99	0.04	-0.03
Global Factor	0.00	1.02		0.00	1.02	

Average spreads are higher in the exporter economy (0.16% per year) than in the importer economy (0.07% per year). This difference arises because default incentives in the exporter economy are more sensitive to the global factor. Fluctuations in the global factor therefore lead to more frequent movements toward the default frontier, increasing average spreads. Consistent with the shape of the default frontier, spreads are procyclical with the global factor in the importer case and countercyclical in the exporter case.

Although mean consumption is similar across the two economies, consumption is much smoother in the exporter case, with volatility an order of magnitude lower than in the importer case. In the importer economy, the price-index and terms-of-trade channels move in the same direction, amplifying the impact of the global factor on consumption. In contrast, in the exporter economy, the two channels offset each other, which dampens the effect of global shocks on consumption.

We now turn to the dynamics of the model around default events. Figure 6 displays the behavior of key variables around default episodes, averaged across all events in the 500 simulations for both economies. In both cases, defaults are triggered by extreme realizations of the global factor. In the importer economy, default events are preceded by large positive shocks to the global factor that generate sharp declines in consumption. In the exporter economy, defaults occur after negative global shocks that reduce export prices and, through the terms-of-trade channel, lower consumption. The decline in consumption is smaller than in the importer case, however, because the price-index channel partly offsets the deterioration in the terms of trade. In both economies, default events are accompanied by sharp increases in sovereign spreads. Both the level and the magnitude of the surge in spreads are larger in the exporter economy, reflecting the higher sensitivity of default incentives to the global factor.

Figure 6: Dynamics Around Default Events.



Note: This figure shows the average dynamics of the key variables around default evens, for both the exporter (orange lines) and importer (blue lines) cases. For both cases, the default events are drawn from 500 simulations of 10,000 periods.

5 Conclusion

This paper provides a unified empirical and quantitative analysis of how global price shocks shape sovereign risk through trade exposure. Empirically, using a shift–share design based on detailed import and export baskets, we show that import price shocks raise sovereign spreads and constrain external borrowing, while export price shocks have weaker and opposite effects. These results reveal that the cyclicality of sovereign spreads depends not only on global financial conditions but also on the composition of trade, which determines how global price movements translate into real income shocks.

Theoretically, we develop a sovereign default model in which global price shocks operate through two structural channels: a *price-index channel*, affecting the cost of living, and a *terms-of-trade income channel*, affecting export revenues. The model endogenously links trade composition to default incentives and replicates the asymmetric behavior of spreads across exporter and importer economies observed in the data. Importantly, the analysis highlights that global price cycles have asymmetric consequences across countries: while exporters benefit from commodity booms through improved terms of trade, import-dependent economies face the opposite dynamic, with rising costs of living and heightened default risk. By grounding the analysis in observable trade exposures, the framework bridges reduced-form evidence and structural modeling, offering a tractable way to study the macro-financial transmission of global shocks.

Taken together, the results highlight that the structure of trade is a key determinant of sovereign resilience to global cycles. Policies that foster export diversification, strengthen fiscal buffers, or hedge against terms-of-trade risk can mitigate the amplification of global price shocks on sovereign borrowing conditions. More broadly, recognizing the asymmetric effects of global price movements across exporters and importers is essential for designing international financial architectures that enhance stability in a world of increasingly synchronized commodity and price cycles.

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Appendix

A Empirical Appendix

Table 6 reports the effect of the shift–share instrument constructed excluding energy commodities on new external debt. Compared with the main specification in Table 2, the results are very similar. In most cases, both import and export price shocks have negative but statistically insignificant effects on new external debt. When the sample is restricted to low-debt or low-spread countries, the estimated effects are close to zero.

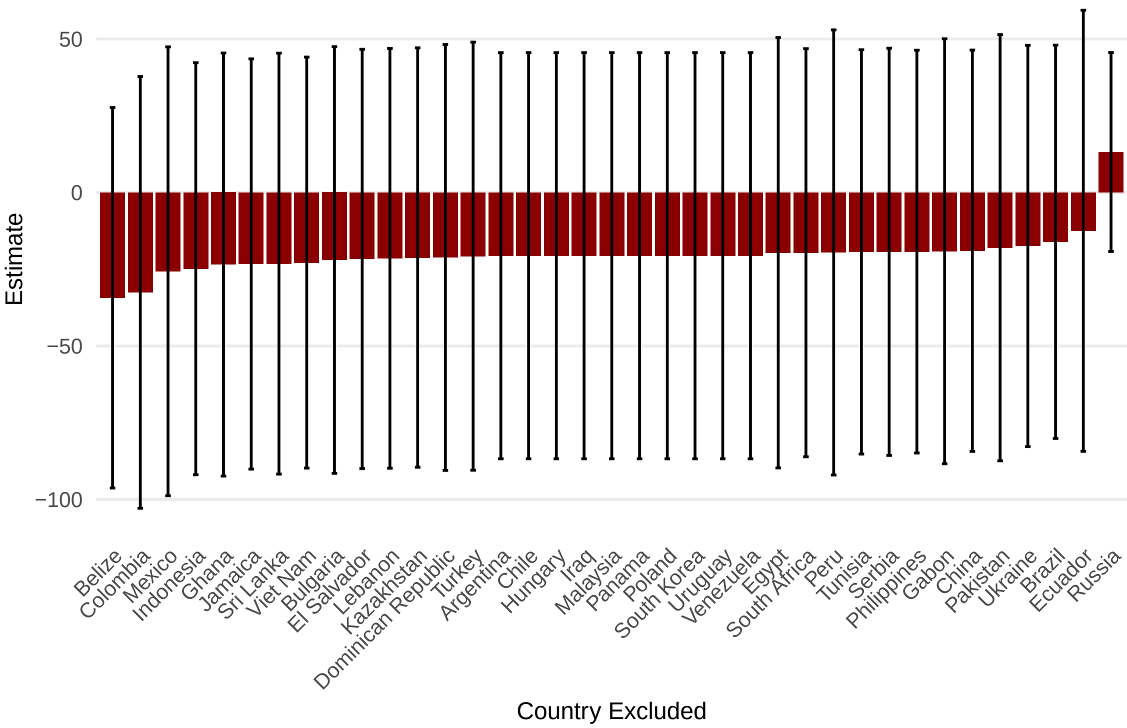
Table 6: Effect on New External Debt, Excluding Energy Commodities.

	Δ debt					
	(1)	(2)	(3)	(4)	(5)	(6)
Import Price Shock	-0.183		-0.292	-0.245	0.035	0.074
	(0.162)		(0.253)	(0.109)	(0.101)	(0.128)
Export Price Shock		-0.194	-0.273	-0.161	-0.083	-0.096
		(0.246)	(0.292)	(0.119)	(0.113)	(0.101)
Observations	755	757	755	753	351	520
R^2	0.37	0.37	0.37	0.62	0.57	0.42
Country fixed effects	✓	✓	✓	✓	✓	✓
Year fixed effects	✓	✓	✓	✓	✓	✓
Extra controls				✓		
Low spreads					✓	
Low debt						✓

Note: This table shows the estimates of import and export price shocks, measured by a shift-share instrument, on the new external debt received by the general government. All regressions control for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. The extra controls include the growth rate of the current account, the average maturity, and the growth rate in interest and principal debt repayments. The 'low spreads' sample restricts changes in spreads to be smaller than 500bp. The 'low debt' sample restricts external debt to output ratio to be smaller than 30%. The shocks are normalized to one standard deviation. Standard errors are clustered at the country level. The shift-share instrument is constructed excluding energy commodities.

To complement the analysis for import price shocks, Figure 7 presents the leave-one-out estimates for export price shocks, based on the same specification and set of controls used in Figure 2.

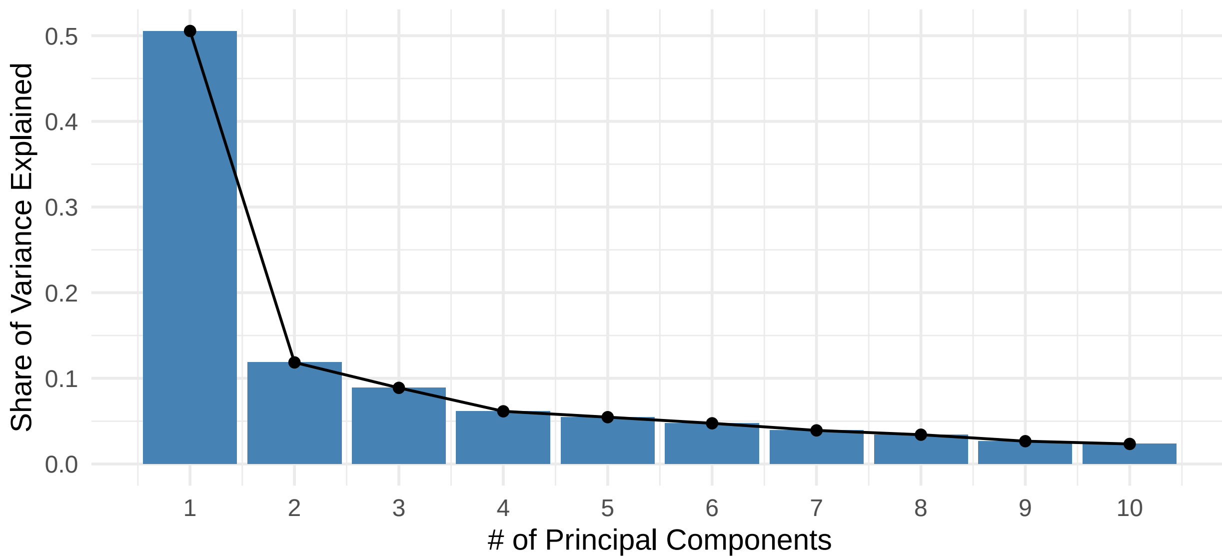
Figure 7: Leave-One-Out Estimates: Export Price Shocks.



Note: This figure shows the effect of export price shocks and sovereign spreads, excluding one country at a time from the sample. Each vertical bar corresponds to the country excluded. The horizontal brackets show the 95% confidence interval. The specification includes both import and export price shocks, and controls for the sum of the shares, the growth rate of real output, the one-year CPI inflation and the nominal exchange rate. Standard errors are clustered at the country level.

Figure 8 shows the share of variance explained by the first K principal components of the log price data for 40 commodities over the period 1992-2020.

Figure 8: Variance Explained by Principal Components.



Note: This figure shows the share of variance explained by the first K principal components of the log price data for 40 commodities over the period 1992-2020.

B Derivations of the Aggregate Environment

This appendix provides the full derivations underlying Section 3.1, including the expenditure minimization problem, the exact price index, and the decomposition of aggregate consumption in factor form.

Price Structure and Factors. The price of each tradable good $i \in [0, 1]$ follows the log-linear factor representation introduced in equation (4):

$$\ln P_t(i) = \mu(i) + a(i)^\top f_t + \varepsilon_t(i),$$

where $f_t \in \mathbb{R}^K$ collects the first K global principal components. The vector of factors evolves according to the stationary VAR(1) process given in equation (5):

$$f_t = \Phi f_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \Sigma_u),$$

with eigenvalues of Φ strictly inside the unit circle.

Preferences and Aggregator. Consumption of each good i is denoted $c_t(i)$, with subsistence level $\bar{c}(i)$. The Stone–Geary aggregator introduced in the main text is:

$$c_t = \prod_{i\in[0,1]} [c_t(i) - \bar{c}(i)]^{\lambda(i)}, \qquad \int_0^1 \lambda(i)\, di = 1,$$

where $\lambda(i) \geq 0$ is the weight assigned to good i. Taking logs yields:

$$\ln c_t = \int_0^1 \lambda(i) \ln[c_t(i) - \bar{c}(i)]\, di.$$

Expenditure Minimization and Individual Demands. Given aggregate expenditure E_t, the representative agent minimizes:

$$\int_0^1 P_t(i) c_t(i)\, di,$$

subject to the aggregator above. The first-order conditions imply:

$$c_t(i) = \bar{c}(i) + \frac{\lambda(i)}{P_t(i)} \Big(E_t - \textstyle\int_0^1 P_t(i')\bar{c}(i')\, di' \Big).$$

The term in parentheses is total expenditure net of subsistence consumption. This expression highlights that, conditional on E_t, additional consumption above subsistence in each good is inversely proportional to its price and weighted by $\lambda(i)$.

Aggregation and the Price Index. Substituting the individual demand functions into the aggregator yields:

$$c_t = \Gamma^c \exp\Big(- \int_0^1 \lambda(i) \ln P_t(i)\, di \Big) \Big(E_t - \int_0^1 P_t(i)\bar{c}(i)\, di \Big),$$

where $\Gamma^c \equiv \prod_{i\in[0,1]} \lambda(i)^{\lambda(i)}$. Define the consumption-based price index:

$$P_t^C \equiv \exp\left\{ \int_0^1 \lambda(i) \ln P_t(i)\, di \right\},$$

so that:

$$c_t = \Gamma^c\, e^{-\ln P_t^C} \Big(E_t - \int_0^1 P_t(i)\bar{c}(i)\, di \Big). \tag{23}$$

Resource Constraint. Define the net endowment $\tilde{e}(i) \equiv e(i) - \bar{c}(i)$. Substituting into the aggregate expenditure identity yields the resource constraint:

$$E_t - \int_0^1 P_t(i)\bar{c}(i)\,di = D_t + \int_0^1 P_t(i)\tilde{e}(i)\,di, \tag{24}$$

which corresponds to equation (6) in the main text.

Price Index in Factor Form. Using the factor structure from equation (4), the consumption-based price index inherits the same factor representation:

$$\ln P_t^C = \mu^C + \psi^\top f_t + \varepsilon_t^C, \tag{25}$$

where:

$$\mu^C = \int_0^1 \lambda(i)\mu(i)\,di, \qquad \psi = \int_0^1 \lambda(i)a(i)\,di, \qquad \varepsilon_t^C = \int_0^1 \lambda(i)\varepsilon_t(i)\,di.$$

The vector ψ summarizes how the aggregate consumption basket responds to global price shocks.

Net-Export Term and Trade Exposure. Let $\bar{P}(i) \equiv e^{\mu(i)}$ denote steady-state prices. A first-order approximation of the trade term around $\bar{P}(i)$ gives:

$$\int_0^1 P_t(i)\tilde{e}(i)\,di \approx \bar{Z} + \beta^\top f_t + \zeta_t,$$

where:

$$\bar{Z} = \int_0^1 \bar{P}(i)\tilde{e}(i)\,di, \qquad \beta_k = \int_0^1 \bar{P}(i)\tilde{e}(i)a_k(i)\,di,$$

and the residual $\zeta_t \equiv \int_0^1 \bar{P}(i)\tilde{e}(i)\varepsilon_t(i)\,di$ is small under diversified trade exposure.

Aggregate Consumption. Substituting (25) and (24) into (23) yields:

$$c_t = \Gamma^c \exp\Big(-\mu^C - \psi^\top f_t - \varepsilon_t^C\Big)\Big(D_t + \bar{Z} + \beta^\top f_t + \zeta_t\Big).$$

Neglecting the small idiosyncratic terms ε_t^C and ζ_t, we obtain the compact form:

$$c_t = \Gamma^c\, e^{-\mu^C - \psi^\top f_t}\Big(D_t + \bar{Z} + \beta^\top f_t\Big), \tag{26}$$

which corresponds exactly to equation (7) in the main text.

C Proofs and Extensions

This appendix collects the analytical proofs of the main theoretical results and presents two extensions. Section C.1 derives the consumption wedge in Proposition 1. Section C.2 characterizes the response of default probabilities and spreads. Finally, Section C.3 establishes sufficient conditions under which these local results extend globally.

C.1 Proof of Proposition 1

Proof. Let $c(f,b)$ be given by:

$$c(f,b) = \Gamma^c \exp(-\mu^C - \psi^\top f)\left(D(f,b) + \bar{Z} + \beta^\top f\right),$$

where $D(f,b)$ is the equilibrium net resource flow implied by the optimal debt choice and the bond pricing function.

Taking logs and differentiating with respect to factor component f_k yields:

$$\frac{\partial \ln c(f,b)}{\partial f_k} = -\psi_k + \frac{\beta_k + \frac{\partial D(f,b)}{\partial f_k}}{D(f,b) + \bar{Z} + \beta^\top f}. \tag{27}$$

Within a period, the government chooses next-period debt b' after observing f. Write:

$$D(f,b) = q(f,b')\,b' - b, \quad b' = b'(f,b) \quad \text{(optimal)}.$$

The derivative $\partial D/\partial f_k$ consists of a "direct" price effect and an "indirect" policy effect via $b'(f,b)$:

$$\frac{\partial D}{\partial f_k} = \underbrace{\frac{\partial q(f,b')}{\partial f_k}\,b'}_{\text{direct}} + \underbrace{\left[q(f,b') + \frac{\partial q(f,b')}{\partial b'}\,b'\right]\frac{\partial b'}{\partial f_k}}_{\text{indirect}}.$$

By the envelope theorem applied to the government's period problem, the indirect term does not contribute to the *marginal* effect of f_k on the objective at the optimum (the bracketed coefficient multiplies the FOC for b'). Consequently, for the *local* impact on c we may evaluate (27) holding b' fixed, which removes the indirect term. In addition, we linearize around a reference point with $f = \bar{f}$ (w.l.o.g. $\bar{f} = \mathbf{0}$ after PCA normalization), so that $\beta^\top f$ is second order in deviations. Under this local approximation,

$$\frac{\partial D(f,b)}{\partial f_k} \approx 0, \qquad D(f,b) + \bar{Z} + \beta^\top f \approx D(f,b) + \bar{Z}.$$

Substituting into (27) gives:

$$\frac{\partial \ln c(f,b)}{\partial f_k} = -\psi_k + \frac{\beta_k}{D(f,b)+\bar{Z}} = -\psi_k + \kappa_c(f,b)\,\beta_k, \quad \kappa_c(f,b) \equiv \frac{1}{D(f,b)+\bar{Z}}.$$

This establishes the expression for the *consumption wedge* $\Delta_k(f,b)$ stated in the proposition.

The sign of $\Delta_k(f,b)$ determines whether a marginal increase in f_k raises or lowers $c(f,b)$: if $\Delta_k > 0$ the terms-of-trade income component $\kappa_c\beta_k$ dominates the price-index drag $-\psi_k$, and vice versa. This yields part (i).

The leverage term $\kappa_c(f,b) = 1/[D(f,b)+\bar{Z}]$ is larger when $D(f,b)+\bar{Z}$ is smaller (i.e., when the economy has weaker external buffers), which magnifies the contribution of β_k to Δ_k. This proves part (ii).

Let $V^r(f,b)$ denote the repayment value. Since $u'(\cdot) > 0$ and the continuation value is increasing in current consumption under standard regularity, an increase in f_k with $\Delta_k(f,b) > 0$ raises $c(f,b)$ and thereby $V^r(f,b)$. Under a monotone default policy, higher V^r weakly reduces the default region and hence the equilibrium default probability; by the lenders' zero-profit condition, this compresses sovereign spreads. The converse holds when $\Delta_k(f,b) < 0$. This proves part (iii).

The key step is the local (impact) derivative evaluated at the optimal b' using the envelope theorem and a first-order expansion around the reference factor state $\bar{f}$. A full dynamic response allowing b' and prices to adjust intertemporally affects higher-order terms but does not alter the stated impact decomposition. □

C.2 Global factors, default sets, and spreads

The next result establishes how the sign of $\Delta_k(f,b)$ translates into equilibrium default probabilities and sovereign spreads extending Proposition 1.

Corollary 1 (Global factors, default sets, and spreads)**.** *Suppose $u'(c) > 0$, $u''(c) \le 0$, and equilibrium repayment is characterized by a single-crossing rule with a cutoff $b^*(f)$ such that the government repays iff $b \ge b^*(f)$. Let bond prices satisfy the zero-profit condition:*

$$q(f,b') \;=\; \frac{1-\delta(f,b')}{1+r^*}, \qquad \delta(f,b') \;=\; \int_{D(b')} f(df' \mid f),$$

where $D(b') = \{\, f' : V^r(f',b') < V^d(f') \,\}$ and $r^* > 0$ is constant.

If, for factor k, the consumption wedge in Proposition 1 satisfies:

$$\Delta_k(f,b) \;=\; -\psi_k + \kappa_c(f,b)\,\beta_k \;>\; 0,$$

then a marginal increase in f_k (holding the other components of f fixed) has the following effects:

(i) ***Default set shrinks.*** *The repayment cutoff weakly decreases, $\partial b^*(f)/\partial f_k \leq 0$; equivalently, the default region $D(\cdot)$ shrinks in measure. Hence the one-step default probability weakly falls: $\partial\delta(f,b')/\partial f_k \leq 0$ for all b'.*

(ii) ***Bond prices rise and spreads compress.*** *Since $q(f,b')$ is decreasing in $\delta(f,b')$, we have $\partial q(f,b')/\partial f_k \geq 0$ and thus the sovereign spread $s(f,b') \equiv 1/q(f,b') - 1 - r^*$ weakly decreases.*

The opposite comparative statics obtain when $\Delta_k(f,b) < 0$.

Proof. Fix (f,b) and perturb factor f_k by a small positive amount. By Proposition 1,

$$\frac{\partial \ln c(f,b)}{\partial f_k} = \Delta_k(f,b).$$

If $\Delta_k(f,b) > 0$, then $\partial c(f,b)/\partial f_k = c(f,b)\,\Delta_k(f,b) > 0$. Since $u'(c) > 0$ and $u''(c) \leq 0$,

$$\frac{\partial}{\partial f_k}\Big[u(c(f,b))\Big] \;=\; u'(c)\,\frac{\partial c}{\partial f_k} \;>\; 0,$$

and the continuation term in V^r is (weakly) increasing in current c by standard monotonicity of the Bellman operator under concavity. Hence:

$$\frac{\partial V^r(f,b)}{\partial f_k} > 0.$$

Under a monotone default policy, a higher V^r (pointwise) weakly reduces the default region, so the next–period default probability decreases. If, locally, the bond price is weakly increasing in repayment value ($\partial q/\partial V^r \geq 0$), then the lenders' zero–profit condition implies q rises and the spread $s = 1/q - r^*$ falls. The converse holds when $\Delta_k(f,b) < 0$. □

C.3 Global comparative statics under bounded leverage

While Proposition 1 and Corollary 1 describe local responses to marginal changes in global factors, many relevant episodes—such as commodity price surges or collapses—entail large and persistent shocks. To assess these nonlinear effects, the following lemma establishes sufficient conditions under which the qualitative results extend globally, even when $D(f,b)$ and default decisions respond endogenously to large movements in f_k.

Lemma 1 (Global comparative statics under bounded leverage)**.** *Fix a nonempty set $\mathcal{S} \subset \mathbb{F} \times \mathbb{B}$ of admissible states and suppose the endogenous resource flow satisfies:*

$$D_{\min} \;\leq\; D(f,b) \;\leq\; D_{\max} \qquad \textit{for all } (f,b) \in \mathcal{S},$$

with $D_{\min} + \bar{Z} > 0$. Let $\kappa_c(f,b) = 1/[D(f,b) + \bar{Z}]$. Then:

(i) ***Exporter dominance.*** *If*

$$\frac{\beta_k}{D_{\max} + \bar{Z}} > \psi_k,$$

then for all $(f,b) \in \mathcal{S}$ the map $f_k \mapsto c(f,b)$ is strictly increasing. Moreover, $V^r(f,b)$ is strictly increasing in f_k, the default set shrinks (in the sense of set inclusion), and bond prices $q(f,b')$ weakly increase.

(ii) ***Importer dominance.*** *If*

$$\frac{\beta_k}{D_{\min} + \bar{Z}} < \psi_k,$$

then for all $(f,b) \in \mathcal{S}$ the map $f_k \mapsto c(f,b)$ is strictly decreasing. Consequently, $V^r(f,b)$ weakly decreases in f_k, the default set expands, and $q(f,b')$ weakly falls.

Proof. Write:

$$c(f,b) \;=\; \Gamma^c\, e^{-\mu^C - \psi^\top f} \Big(D(f,b) + \bar{Z} + \beta^\top f \Big),$$

and note that $D(f,b) + \bar{Z} > 0$ on $\mathcal{S}$ by assumption ($D_{\min} + \bar{Z} > 0$) and that $c(\cdot)$ is strictly positive.

Step 1 (A uniform sign bound for the marginal effect on c). Consider the "mechanical" (Gateaux) effect of f_k on $\ln c$ holding the policy D fixed:

$$\Big(\partial_{f_k} \ln c \Big)\Big|_D \;=\; -\psi_k \;+\; \frac{\beta_k}{D(f,b) + \bar{Z} + \beta^\top f}.$$

Since $D(f,b) \in [D_{\min}, D_{\max}]$ on $\mathcal{S}$ and $D(f,b) + \bar{Z} > 0$, we have:

$$\frac{1}{D(f,b) + \bar{Z} + \beta^\top f} \;\geq\; \frac{1}{D(f,b) + \bar{Z}} \;\geq\; \frac{1}{D_{\max} + \bar{Z}} \quad \text{if } \beta_k > 0,$$

and:

$$\frac{1}{D(f,b) + \bar{Z} + \beta^\top f} \;\leq\; \frac{1}{D(f,b) + \bar{Z}} \;\leq\; \frac{1}{D_{\min} + \bar{Z}} \quad \text{if } \beta_k < 0,$$

because increasing the denominator weakens (in absolute value) the fraction and $x \mapsto 1/x$ is decreasing on $(0, \infty)$. Therefore,

$$\Big(\partial_{f_k} \ln c \Big)\Big|_D \;\geq\; -\psi_k + \frac{\beta_k}{D_{\max} + \bar{Z}} \quad \text{if } \beta_k > 0, \qquad \Big(\partial_{f_k} \ln c \Big)\Big|_D \;\leq\; -\psi_k + \frac{\beta_k}{D_{\min} + \bar{Z}} \quad \text{if } \beta_k < 0.$$

Step 2 (Exporter dominance). If $\beta_k/(D_{\max}+\bar{Z}) > \psi_k$, then by Step 1:

$$\left(\partial_{f_k} \ln c\right)\Big|_D \; > \; 0 \quad \text{for all } (f,b) \in \mathcal{S}.$$

Because u is increasing and the repayment Bellman operator is monotone in c, this strictly raises the repayment value $V^r(f,b)$. Hence $V^r(f,b) - V^d(f)$ strictly increases in f_k, the default set shrinks in the sense of set inclusion, and lenders' zero-profit prices $q(f,b')$ weakly rise (default probabilities weakly fall). This proves (i).

Step 3 (Importer dominance). If $\beta_k/(D_{\min}+\bar{Z}) < \psi_k$, then by Step 1:

$$\left(\partial_{f_k} \ln c\right)\Big|_D \; < \; 0 \quad \text{for all } (f,b) \in \mathcal{S}.$$

The same monotonicity logic implies $V^r(f,b)$ weakly decreases in f_k, the default set expands, and $q(f,b')$ weakly falls. This proves (ii).

Remarks. (a) The bounds are *uniform* on $\mathcal{S}$ because they only use $D(f,b) \in [D_{\min}, D_{\max}]$ and $D_{\min} + \bar{Z} > 0$. (b) We evaluated the direct (mechanical) effect holding D fixed; the endogenous feedback of D through policies cannot overturn the sign because the repayment problem is monotone in c and lenders price competitively off default probabilities. Thus the conditions stated are sufficient (global) sign conditions. □

D Estimation and Solution Algorithm

This appendix describes the empirical construction of the exposure parameters $(\psi, \beta, \bar{Z}, \mu^C)$ and the numerical procedure used to compute the equilibrium. The structure mirrors the calibration framework in Section 4.1. We first estimate the global factor structure and exposure parameters from the data, and then, conditional on these inputs, solve for the government's optimization problem and the corresponding market-clearing bond price schedule.

Link to PCA Facts. The factorization:

$$\ln P_t(i) = \mu(i) + a(i)^\top f_t + \varepsilon_t(i)$$

mirrors the empirical evidence presented in Section 2. The first principal component (PC1) explains approximately 65% of the total variance of international prices, and the first six account for about 90%. The persistence of global shocks is captured by the process:

$$f_t = \Phi f_{t-1} + u_t, \qquad u_t \sim \mathcal{N}(0, \Sigma_u),$$

with eigenvalues of Φ strictly inside the unit circle. Idiosyncratic components $\varepsilon_t(i)$ are near white noise, consistent with the empirical autocorrelations after purging principal components.

Two Exposure Vectors. Aggregate sensitivity to global shocks is governed by two exposure vectors:

$$\text{(i) Price-index channel: } -\psi^\top f_t, \qquad \text{(ii) Terms-of-trade channel: } +\beta^\top f_t.$$

Export concentration in goods with high factor loadings $a_k(i)$ raises β_k; concentration of consumption weights $\lambda(i)$ where $a_k(i)$ is large raises ψ_k.

Calibration/Estimation Steps.

1. **Principal Component Estimation.** Let N be the number of commodities (here, $N = 40$). Construct the $T \times N$ matrix of demeaned log prices:

$$X = \left[\ln P_{t,i} - \ln \bar{P}_i\right]_{t=1,\ldots,T;\, i=1,\ldots,40}.$$

   Run PCA on X (covariance PCA) to obtain the factor representation:

$$X \approx FA^\top, \qquad F \in \mathbb{R}^{T\times K}, \;\; A \in \mathbb{R}^{40\times K},$$

   where the ith row of A, denoted $a(i)^\top$, contains the K loadings for good i so that $\ln P_{t,i} \approx \ln \bar{P}_i + a(i)^\top f_t$. Here $f_t \in \mathbb{R}^K$ is the $K \times 1$ vector of factor realizations at time t, and $\bar{P}_i \equiv \exp(\overline{\ln P_i})$ is the baseline price of good i. The PCA factors are normalized to have zero mean and identity covariance, i.e. $\mathbb{E}[f_t] = 0$ and $\mathbb{E}[f_t f_t^\top] = I_K$.

2. **Factor Dynamics.** Estimate by OLS the autoregressive process:

$$f_t = \Phi f_{t-1} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma_u),$$

   and discretize the process into a finite Markov chain $\{f^j,\ P_f(j,j')\}$ using the quadrature method of Tauchen & Hussey (1991).

3. **Inputs from Consumption and Trade Data.** Collect (i) the vector of consumption weights $\lambda = (\lambda_1, \ldots, \lambda_{40})$ with $\sum_i \lambda_i = 1$; (ii) the net endowments quantities $\tilde{e} = (\tilde{e}_1, \ldots, \tilde{e}_{40})$ defined by $\tilde{e}(i) = e(i) - \bar{c}(i)$; and (iii) the baseline prices $\bar{P}_i = e^{\mu(i)}$. These three objects map the observed structure of consumption and trade into model parameters.

4. **Computation of Exposure Parameters.** The exposure vectors and constants are

computed as:

$$\psi = A^\top \lambda, \qquad \beta = A^\top(\bar{P} \odot s),$$
$$\bar{Z} = \sum_{i=1}^{40} \bar{P}_i s_i, \qquad \mu^C = \sum_{i=1}^{40} \lambda_i \mu(i),$$

where $\odot$ denotes elementwise multiplication. The pair (ψ, β) fully captures the price-index and terms-of-trade channels in the aggregate consumption function, while $(\bar{Z}, \mu^C)$ determine the steady-state trade balance and the average consumption price index.

5. **Country Profiles.** We consider two economies that differ only in their trade structure:

   - **Exporter:** $\tilde{e}(i)$ concentrated in goods with high positive loadings $a(i) > 0$, implying $\beta > 0$.
   - **Importer:** $\tilde{e}(i)$ concentrated in goods with negative loadings $a(i) < 0$, implying $\beta < 0$.

   The consumption exposure ψ is similar across both economies, reflecting a diversified consumption basket.

6. **Solving the Model.** Given the empirical objects $(\psi, \beta, \bar{Z}, \mu^C)$ and the calibrated parameters, we solve for the government's value and policy functions as follows:

   (a) Set an initial bond price schedule $q^{(0)}(f, b') = 1/(1 + r^*)$ and initialize the value functions $V^{r,(0)}(f, b)$ and $V^{d,(0)}(f)$.

   (b) For each state (f, b), compute the repayment value:

   $$V^r(f, b) = \max_{b'} \left\{ u(c^r(f, b, b')) + \beta \sum_{f'} P_f(f, f') V(f', b') \right\},$$

   with

   $$c^r(f, b, b') = \Gamma^c e^{-\mu^C - \psi f} \left[q(f, b')b' - b + \bar{Z} + \beta f\right].$$

   The default value is:

   $$V^d(f) = u(c^d(f)) + \beta \sum_{f'} P_f(f, f')[\theta V(f', 0) + (1 - \theta)V^d(f')],$$

   where

   $$c^d(f) = \chi \, \Gamma^c e^{-\mu^C - \psi f} \, (\bar{Z} + \beta f).$$

   (c) Update $V(f, b) = \max\{V^r(f, b), V^d(f)\}$ and identify the default region $D(b)$ where $V^d(f) > V^r(f, b)$.

   (d) Compute the default probabilities $\delta(f, b') = \sum_{f' \in D(b')} P_f(f, f')$ and update bond prices using

   $$q^{(1)}(f, b') = \frac{1 - \delta(f, b')}{1 + r^*}.$$

(e) Iterate until convergence of the bond price function and the value functions.

At convergence, the equilibrium policies, prices, and default probabilities jointly satisfy government optimality and market-clearing.

Summary. Steps (1)–(5) discipline the model empirically by estimating the exposure parameters from the international price data, while Step (6) solves the model numerically through value function iteration. The resulting equilibrium captures how the interaction between global price shocks and trade exposure shapes consumption, debt, and default risk. In all quantitative exercises that follow, we set $K = 1$.